On Labelling Statements in Multi-labelling Argumentation

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Abstract. In computational models of argumentation, argument justification has attracted more attention than statement justification, and significant sensitivity losses are identifiable when dealing with the justification of statements by otherwise appealing formalisms. This paper reappraises statement justification as a formalism-independent component in argument-based reasoning. We introduce a novel general model of argument-based reasoning based on multiple stages of labellings, the last one being devoted to statement justification, identify two alternative paths from argument acceptance to statement justification, and compare their expressiveness. We then show that this model encompasses several prominent literature proposals as special cases, thereby enabling a systematic comparison of existing approaches to statement justification, evidencing their merits and limits. Finally we illustrate our model by specifying a generic ignorance-aware statement justification and showing how it can be seamlessly integrated into different formalisms.

1 INTRODUCTION

In studies of argument-based reasoning, argument justification has received far more attention than statement justification, often treated as a simple byproduct of the former. As a consequence, significant expressiveness and sensitivity problems can be identified in the treatment of statement justification by otherwise appealing formalisms. In particular, in a recent paper [4] we have pointed out that even in very simple common sense reasoning examples the statement justification outcomes produced by different argumentation formalisms may be significantly different and show counterintuitive aspects. To overcome these limitations, in [4] we have proposed a preliminary approach where statement justification is regarded as a formalism-independent tunable component of argument-based reasoning. The approach is based on a generic multi-labelling system and its application and relevant advantages have been exemplified in the case of $ASPIC^+$ [11]. In this paper we provide a twofold advancement in this research direction. First we identify two alternative design choices in multi-labelling systems and compare their expressiveness. Second we illustrate the application of multi-labelling systems to model a representative set of argumentation formalisms, thus confirming their potential to provide a common formalism-independent reference for the investigation of statement justification principles and methods. The presentation is supported by a common sense example, illustrated below.

Example 1. *Suppose that Dr. Smith says to you: "Given your clinical picture, you are affected by disease D1, not disease D2". Suppose then Dr. Jones, considered equally competent, says to you: "Given your clinical picture, you are affected by disease D2, not by disease D1". Your view on the justification of the statements S1="I am affected by disease D1" and S2="I am affected by disease D2" may become quite uncertain. In a different situation, at home, you use an off-the-shelf test kit suggesting you have caught disease D3. You then undertake a serious and reliable clinical test, which excludes disease D3. Would you consider the same status for the statement S3="I am affected by disease D3" and the statements S1, S2? What about the justification status of the statement S4="I am affected by D4", where D4 is a poorly studied and initially asymptomatic disease you never heard of? Intuitively, different justification statuses seem reasonable and useful. Actually, such distinctions may be decisive. Surprisingly, as will be discussed in the following, the current versions of several well-known structured argumentation formalisms fail to distinguish these justification statuses, equating, for instance, the justification status of S4 with the one of S3, or with that of S1 and S2, or even the justification status of S3 with that of S1 and S2.*

We advocate that the loss of sensitivity to statements' statuses is not intrinsic to the formalisms, but is rather due to the relatively limited attention paid to justification of statements, often treated as a sort of appendix of the notions of acceptance and justification of arguments. To address this limitation, we propose a multi-labelling model of the argumentation process, showing that starting from the common basis of argument production and acceptance two alternative approaches to statement justification can be considered.

The paper is organised as follows. In Section 2 we propose multi-labelling systems for argumentation, catering for an argument-focused approach and a statement-focused approach. In Section 3 we compare the expressiveness of the two approaches. We show in Section 4 that several literature proposals can be seen as instances of our model, and in Section 5 how it can support tunable statement justification labellings, before concluding in Section 6.

2 MULTI-LABELLING SYSTEMS

To investigate the different notions of justification involved in a generic argument-based reasoning system, one may define a generic multi-labelling model of the argumentation process. In [4] we preliminarily introduced a model consisting of four stages (or levels), namely argument production, argument acceptance, argument justification, statement justification. Here we extend our analysis with two variants of the model, called the *argument-focused* approach and the *statement-focused* approach. These two approaches differ at the third stage (see Fig. 1). In the *argument-focused* approach, argument acceptance gives rise to argument justification at a third stage, from which statement justification is derived at a fourth stage.

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In the *statement-focused* approach, argument acceptance is projected on statements, giving rise to statement acceptance at a third stage, from which statement justification is again derived at a fourth stage. The description and formal definitions of these different stages are provided in the sequel, preceded by some basic concepts. Due to space limits we cannot include illustrative examples in this section: they are provided with the analyses developed in Sections 3 and 4.

Basic concepts. Multi-labelling systems are based on the notion of labelling.

Definition 1 (Labelling)**.** *Given a set of labels* Λ *and a set* T*, a* Λ*-labelling* L *of* T *is a (possibly partial) function* $L : T \to \Lambda$.

The idea is then that argument-based reasoning can be regarded as a sequence of labelling activities where the starting point consists in producing labellings for arguments and the final result is a labelling for the statements about which the arguments are built. Moving across the stages, the labellings produced at one stage are used as input to produce new labellings at the next stage, where the labels, their meaning and/or the labelled elements change. As we will see, it may happen that sets of labels are projected or transferred from the elements of a stage to those in the subsequent one (e.g. a statement may "receive" the set of labels associated to all the arguments concluding it). In these cases a synthesis (S) operator is required.

Definition 2 (S-operators)**.** *Given two sets of labels* Λ_1 *and* Λ_2*, a S-operator from* Λ_1 *to* Λ_2 *is a function* $\mathfrak{S} : Pow(\Lambda_1) \to \Lambda_2$*, where* $Pow(T)$ *denotes the powerset of* T*. A double S-operator from* Λ_1 *to* Λ_2 *is a function* $\mathfrak{S} : Pow(\Lambda_1) \times Pow(\Lambda_1) \to \Lambda_2$.

Intuitively, a synthesis operator associates with each set of labels in Λ_1 a single corresponding label in Λ_2. Such an operator is naturally applied to projected or transferred sets of labels belonging to Λ_1 to obtain a synthetic representation in the context of Λ_2. Double synthesis operators will have a role when contraries come into play.

Argument production. The first stage regards the production of a set of arguments $\mathcal{A}$ whose structure and mutual relationships are left unspecified. The only relevant property for our purposes is that each argument $A \in \mathcal{A}$ has a conclusion, denoted as $Con(A)$, belonging to a language $\mathcal{L}$. We do not make any assumption on the set of arguments, while we assume that the language is equipped with a contrariness relation. In its simplest form the contrariness relation corresponds to the traditional notion of negation but other more general forms of contrariness have been considered in the literature [10, 3]. To encompass this wider view, we assume a contrariness relation Cnt, allowing the existence of multiple (or no) contraries for each statement, and hence being compatible with a variety of argumentation formalisms.

Definition 3 (Language)**.** *A language* $\mathcal{L}$ *is a set of statements equipped with a contrariness relation* $Cnt : \mathcal{L} \to Pow(\mathcal{L})$*. For all* $\varphi \in \mathcal{L}$*,* $\psi \in Cnt(\varphi)$ *is called a contrary of* φ.

The outcomes of the argument production stage can be summarised in an abstract form as an argument-conclusion structure.

Definition 4 (Argument-conclusion structure)**.** *An* argument-conclusion structure *(ACS) is a triple* $\langle \mathcal{L}, \mathcal{A}, Con \rangle$ *where* $\mathcal{L}$ *is a language,* $\mathcal{A}$ *is a finite set of arguments and* $Con : \mathcal{A} \to \mathcal{L}$ *is a relation associating every argument with its conclusion.*

Note that some elements of $\mathcal{L}$ may not play the role of conclusions, e.g. if $\mathcal{L}$ encompasses negation as failure.

In general each statement $\varphi \in \mathcal{L}$ is supported by a (possibly empty) set of arguments denoted as $Arg(\varphi)$. This notion can obviously be extended to sets of statements as in the following definition.

Definition 5 (Supporting arguments)**.** *Given an ACS* $\langle \mathcal{L}, \mathcal{A}, Con \rangle$ *and a set* $\Phi \subseteq \mathcal{L}$*, the set of* supporting arguments *of* Φ *is defined as*

$$Arg(\Phi) \triangleq \{A \in \mathcal{A} \mid Con(A) \in \Phi\}.$$

Argument acceptance. The second stage concerns the acceptance evaluation of a set of arguments, the outcome is a set of argument acceptance labellings using a set of labels Λ_{AA}. Each label in Λ_{AA} represents an individual argument acceptance status and a labelling L_{AA} altogether represents a "reasonable" viewpoint (in general among many possible ones) about the acceptance of the arguments in $\mathcal{A}$.

Definition 6 (Argument acceptance labelling and evaluation)**.** *Given an ACS* $\mathcal{AC} = \langle \mathcal{L}, \mathcal{A}, Con \rangle$ *and a set of acceptance labels* Λ_{AA}*, an* argument acceptance Λ_{AA}-labelling *for* $\mathcal{AC}$ *is a* Λ_{AA}*-labelling of* $\mathcal{A}$.

A Λ_{AA}-acceptance evaluation *for* $\mathcal{AC}$ *is a set of argument acceptance* Λ_{AA}*-labellings for* $\mathcal{AC}$ *denoted as* $\mathfrak{L}_{\mathsf{AA}}(\mathcal{AC})$ *or just* $\mathfrak{L}_{\mathsf{AA}}$ *where not ambiguous.*

Different ways of using the set $\mathfrak{L}_{\mathsf{AA}}$ give rise to two alternatives for the subsequent stages. In a nutshell, in the *argument-focused* approach, the set of acceptance labelling is projected on arguments and then synthesised, giving rise to an argument justification stage, while in the *statement-focused* approach, the focus is transferred from arguments to their conclusions, giving rise to a statement acceptance stage.

Argument-focused (AF) approach

Argument justification. In the AF approach, the third stage deals with the definition of an argument justification labelling L_{AJ} using a set Λ_{AJ} of argument justification labels.

It is natural to assume that L_{AJ} is functionally dependent on $\mathfrak{L}_{\mathsf{AA}}$ and, in particular, we make two basic assumptions on the nature of this dependency. First, for each argument A, $L_{\mathsf{AJ}}(A)$ depends only on the acceptance labels of A in $\mathfrak{L}_{\mathsf{AA}}$; second, cardinality does not count in this evaluation, i.e. for each label $\lambda \in \Lambda_{\mathsf{AA}}$ it only matters whether there is any $L_{\mathsf{AA}} \in \mathfrak{L}_{\mathsf{AA}}$ such that $L_{\mathsf{AA}}(A) = \lambda$. Following these assumptions, L_{AJ} is obtained by first projecting $\mathfrak{L}_{\mathsf{AA}}$ on arguments and then applying a synthesis operator from Λ_{AA} to Λ_{AJ}.

Definition 7 (Argument acceptance projection)**.** *Let* $\mathcal{AC} = \langle \mathcal{L}, \mathcal{A}, Con \rangle$ *be an ACS and* $\mathfrak{L}_{\mathsf{AA}}$ *a* Λ_{AA}*-acceptance evaluation for* $\mathcal{AC}$*. For every* $A \in \mathcal{A}$ *the* projection *of* $\mathfrak{L}_{\mathsf{AA}}$ *on* A *is defined as*

$$\Sigma_{\mathsf{AA}}(A) \triangleq \{\lambda \in \Lambda_{\mathsf{AA}} \mid \exists L_{\mathsf{AA}} \in \mathfrak{L}_{\mathsf{AA}} : L_{\mathsf{AA}}(A) = \lambda\}.$$

Definition 8 (Argument justification labelling and evaluation)**.** *Given a set of justification labels* Λ_{AJ} *and an ACS* $\mathcal{AC} = \langle \mathcal{L}, \mathcal{A}, Con \rangle$*, an* argument justification Λ_{AJ}-labelling *for* $\mathcal{AC}$ *is a* Λ_{AJ}*-labelling of* $\mathcal{A}$*. Given a* Λ_{AA}*-acceptance evaluation* $\mathfrak{L}_{\mathsf{AA}}$ *for* $\mathcal{AC}$*, an argument justification* Λ_{AJ}*-labelling* L_{AJ} *is the synthesis of the projection of* $\mathfrak{L}_{\mathsf{AA}}$ *based on a S-operator* $\mathfrak{S}_{\mathsf{AJ}}$ *from* Λ_{AA} *to* Λ_{AJ} *if for every argument* $A \in \mathcal{A}$ *it holds that* $L_{\mathsf{AJ}}(A) = \mathfrak{S}_{\mathsf{AJ}}(\Sigma_{\mathsf{AA}}(A))$.

AF statement justification. The fourth stage in the AF approach caters for the justification status of statements, i.e. the elements of the language $\mathcal{L}$. We assume that this is functionally dependent on a given argument justification labelling, as defined above, and is represented by exactly one statement justification labelling L_{ASJ} using a set of statement justification labels Λ_{SJ}. We assume that the contraries of a statement may play a role in the assessement of its justification and of course that L_{ASJ} depends on the outcome of the third stage: for each statement φ the justification labels are transferred from arguments to φ itself and to its contraries, yielding $\Upsilon_{\mathsf{AJ}}(\varphi)$ and $\Upsilon_{\overline{\mathsf{AJ}}}(\varphi)$ according to the following definition.

Definition 9 (Justification transfer). *Let $\mathcal{AC} = \langle \mathcal{L}, \mathcal{A}, Con \rangle$ be an ACS and L_{AJ} an argument justification Λ_{AJ}-labelling for $\mathcal{AC}$. For every statement $\varphi \in \mathcal{L}$ the* supporting transfer *and* contrary-supporting transfer *of L_{AJ} on φ are respectively defined as*

$$\Upsilon_{\mathsf{AJ}}(\varphi) \triangleq \{\lambda \in \Lambda_{\mathsf{AJ}} \mid \exists A \in Arg(\{\varphi\}) : L_{\mathsf{AJ}}(A) = \lambda\}$$

$$\Upsilon_{\overline{\mathsf{AJ}}}(\varphi) \triangleq \{\lambda \in \Lambda_{\mathsf{AJ}} \mid \exists A \in Arg(Cnt(\varphi)) : L_{\mathsf{AJ}}(A) = \lambda\}.$$

Based on $\Upsilon_{\mathsf{AJ}}(\varphi)$ and $\Upsilon_{\overline{\mathsf{AJ}}}(\varphi)$, assigning a justification label in Λ_{SJ} to each statement amounts to define a double S-operator $\mathfrak{S}_{\mathsf{ASJ}} : Pow(\Lambda_{\mathsf{AJ}}) \times Pow(\Lambda_{\mathsf{AJ}}) \to \Lambda_{\mathsf{SJ}}$.

Definition 10 (AF statement justification labelling and evaluation). *Given an ACS $\mathcal{AC} = \langle \mathcal{L}, \mathcal{A}, Con \rangle$ and a set of statement justification labels Λ_{SJ}, an* AF statement justification Λ_{SJ}-labelling *for $\mathcal{AC}$ is a Λ_{SJ}-labelling of $\mathcal{L}$. Given an argument justification Λ_{AJ}-labelling L_{AJ} for $\mathcal{AC}$, an AF statement justification Λ_{SJ}-labelling L_{ASJ} is the synthesis of the transfer of L_{AJ} based on a double S-operator $\mathfrak{S}_{\mathsf{ASJ}}$ from Λ_{AJ} to Λ_{SJ} if for every statement $\varphi \in \mathcal{L}$ $L_{\mathsf{ASJ}}(\varphi) = \mathfrak{S}_{\mathsf{ASJ}}(\Upsilon_{\mathsf{AJ}}(\varphi), \Upsilon_{\overline{\mathsf{AJ}}}(\varphi))$.*

Statement-focused (SF) approach

Statement acceptance. Since multiple arguments may have the same conclusion, within each single labelling $L_{\mathsf{AA}} \in \mathfrak{L}_{\mathsf{AA}}(\mathcal{AC})$ one can transfer the acceptance labels from arguments to statements and then synthesise them to obtain a statement acceptance labelling. In this way, a set of statement acceptance labellings can be derived from an argument acceptance evaluation.

Definition 11 (Acceptance transfer). *Let $\mathcal{AC} = \langle \mathcal{L}, \mathcal{A}, Con \rangle$ be an ACS and L_{AA} an argument acceptance Λ_{AA}-labelling for $\mathcal{AC}$. For every statement $\varphi \in \mathcal{L}$ the supporting transfer and contrary-supporting transfer of L_{AA} on φ are respectively defined as*

$$\Upsilon_{\mathsf{AA}}(\varphi) \triangleq \{\lambda \in \Lambda_{\mathsf{AA}} \mid \exists A \in Arg(\{\varphi\}) : L_{\mathsf{AA}}(A) = \lambda\}$$

$$\Upsilon_{\overline{\mathsf{AA}}}(\varphi) \triangleq \{\lambda \in \Lambda_{\mathsf{AA}} \mid \exists A \in Arg(Cnt(\varphi)) : L_{\mathsf{AA}}(A) = \lambda\}.$$

Definition 12 (Statement acceptance labelling and evaluation). *Given a set of statement acceptance labels Λ_{SA} and an ACS $\mathcal{AC} = \langle \mathcal{L}, \mathcal{A}, Con \rangle$, a* statement acceptance Λ_{SA}-labelling *for $\mathcal{AC}$ is a Λ_{SA}-labelling of $\mathcal{L}$. Given an argument acceptance Λ_{AA}-labelling L_{AA} for $\mathcal{AC}$, a statement acceptance Λ_{SA}-labelling L_{SA} is the synthesis of the transfer of L_{AA} based on a double S-operator $\mathfrak{S}_{\mathsf{SA}}$ from Λ_{AA} to Λ_{SA} if for every $\varphi \in \mathcal{L}$ $L_{\mathsf{SA}}(\varphi) = \mathfrak{S}_{\mathsf{SA}}(\Upsilon_{\mathsf{AA}}(\varphi), \Upsilon_{\overline{\mathsf{AA}}}(\varphi))$.*

A statement acceptance evaluation *for $\mathcal{AC}$ is a set of statement acceptance labellings for $\mathcal{AC}$ denoted $\mathfrak{L}_{\mathsf{SA}}(\mathcal{AC})$ or just $\mathfrak{L}_{\mathsf{SA}}$ where not ambiguous.*

SF statement justification. In the SF approach, the statement acceptance evaluation is projected on each statement and on its contraries.

Definition 13 (Statement acceptance projection). *Let $\mathcal{AC} = \langle \mathcal{L}, \mathcal{A}, Con \rangle$ be an ACS and $\mathfrak{L}_{\mathsf{SA}}$ a statement acceptance evaluation for $\mathcal{AC}$. For every statement $\varphi \in \mathcal{L}$ the* projection *of $\mathfrak{L}_{\mathsf{SA}}$ on φ and on its contraries are respectively defined as*

$$\Sigma_{\mathsf{SA}}(\varphi) \triangleq \{\lambda \in \Lambda_{\mathsf{SA}} \mid \exists L_{\mathsf{SA}} \in \mathfrak{L}_{\mathsf{SA}} : L_{\mathsf{SA}}(\varphi) = \lambda\}$$

$$\Sigma_{\overline{\mathsf{SA}}}(\varphi) \triangleq \{\lambda \in \Lambda_{\mathsf{SA}} \mid \exists \overline{\varphi} \in Cnt(\varphi), \exists L_{\mathsf{SA}} \in \mathfrak{L}_{\mathsf{SA}} : L_{\mathsf{SA}}(\overline{\varphi}) = \lambda\}.$$

Then, for each statement φ, a statement justification labelling L_{SSJ} is a function of the acceptance labels of φ itself, and its contraries.

Definition 14 (SF statement justification labelling and evaluation). *Given an ACS $\mathcal{AC} = \langle \mathcal{L}, \mathcal{A}, Con \rangle$ and a set of statement justification labels Λ_{SJ}, a* SF statement justification Λ_{SJ}-labelling *for $\mathcal{AC}$ is a Λ_{SJ}-labelling of $\mathcal{L}$. Given a statement acceptance evaluation $\mathfrak{L}_{\mathsf{SA}}$ for $\mathcal{AC}$, a SF statement justification Λ_{SJ}-labelling L_{SSJ} is the synthesis of the projection of $\mathfrak{L}_{\mathsf{SA}}$ based on a double S-operator $\mathfrak{S}_{\mathsf{SSJ}}$ from Λ_{SA} to Λ_{SJ} if for every statement $\varphi \in \mathcal{L}$ $L_{\mathsf{SSJ}}(\varphi) = \mathfrak{S}_{\mathsf{SSJ}}(\Sigma_{\mathsf{SA}}(\varphi), \Sigma_{\overline{\mathsf{SA}}}(\varphi))$.*

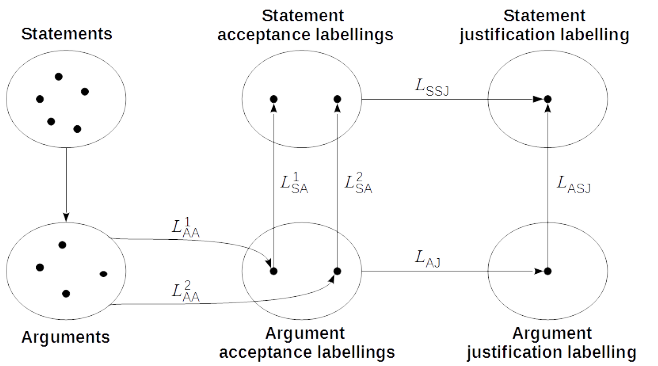

Figure 1. Overview of a multi-labelling system.

A *multi-labelling system* (MLS) consists of an ACS equipped with the relevant evaluations either in the AF or SF approach. Accordingly, two classes of *multi-labelling systems* can be identified:

- an AF MLS is a tuple $\mathfrak{L}^A = \langle \mathcal{AC}, \mathfrak{L}_{\mathsf{AA}}, L_{\mathsf{AJ}}, L_{\mathsf{ASJ}} \rangle$;
- an SF MLS is a tuple $\mathfrak{L}^S = \langle \mathcal{AC}, \mathfrak{L}_{\mathsf{AA}}, \mathfrak{L}_{\mathsf{SA}}, L_{\mathsf{SSJ}} \rangle$;

where all the symbols are interpreted as in the previous definitions.

3 COMPARING THE AF AND SF APPROACHES

In the AF approach the idea is that the outcomes of the second stage (i.e. the acceptance labellings of arguments) are first projected and synthesised on the argument themselves, giving rise to argument justification. Then argument justification outcomes are transferred to statements and synthesised in turn, taking contraries into account, to get statement justification. On the other hand in the SF approach the outcomes of the argument acceptance stage are immediately transferred and synthesised on statements, giving rise to statement acceptance. Then the acceptance outcomes of a statement and of its contraries are taken into account to derive statement justification.

One may wonder whether, under the assumptions we made, the two approaches feature the same expressiveness, i.e. whether any statement justification labelling produced by an AF MLS can be obtained by a corresponding SF MLS and vice versa.

We show that the answer is negative, using some simple examples with the set of argument acceptance labels $\Lambda_{\mathsf{AA}} = \{\mathsf{IN}, \mathsf{OUT}\}$.

A distinction expressible only by the SF approach.

C1. Consider a first case C1 where there are (possibly among others) two arguments A and B such that, for some statement φ, $Arg(\{\varphi\}) = \{A, B\}$ (i.e. they have the same conclusion φ and no other arguments conclude φ). For simplicity, let us also assume that $Arg(Cnt(\varphi)) = \emptyset$. Suppose that the outcome of the argument acceptance stage consists of two labellings, i.e. $\mathfrak{L}_{\mathsf{AA}}(\mathcal{AC}) = \{L^1_{\mathsf{AA}}, L^2_{\mathsf{AA}}\}$ such that $L^1_{\mathsf{AA}}(A) = \mathsf{IN}$ and $L^1_{\mathsf{AA}}(B) = \mathsf{OUT}$, while $L^2_{\mathsf{AA}}(A) = \mathsf{OUT}$ and $L^2_{\mathsf{AA}}(B) = \mathsf{IN}$.

- In the AF approach, at the argument justification stage, we get $\Sigma_{\mathsf{AA}}(A) = \Sigma_{\mathsf{AA}}(B) = \{\mathsf{IN}, \mathsf{OUT}\}$ and, whatever S-operator $\mathfrak{S}_{\mathsf{AJ}}$ is adopted it must be that $L_{\mathsf{AJ}}(A) = L_{\mathsf{AJ}}(B) = \mathfrak{S}_{\mathsf{AJ}}(\{\mathsf{IN}, \mathsf{OUT}\}) = \lambda$ for some $\lambda \in \Lambda_{\mathsf{AJ}}$. At the statement justification stage, we have $\Upsilon_{\mathsf{AJ}}(\varphi) = \{\lambda\}$ and $\Upsilon_{\overline{\mathsf{AJ}}}(\varphi) = \emptyset$. Then, whatever S-operator $\mathfrak{S}_{\mathsf{ASJ}}$ is adopted, we get that $L_{\mathsf{ASJ}}(A)$ functionally depends on the pair $(\{\lambda\}, \emptyset)$ i.e. $L_{\mathsf{ASJ}}(\varphi) = \mathfrak{S}_{\mathsf{ASJ}}(\{\lambda\}, \emptyset)$.
- In the SF approach, at the statement acceptance stage, we get $\Upsilon^1_{\mathsf{AA}}(\varphi) = \Upsilon^2_{\mathsf{AA}}(\varphi) = \{\mathsf{IN}, \mathsf{OUT}\}$. Therefore, $L^1_{\mathsf{SA}}(\varphi) = L^2_{\mathsf{SA}}(\varphi) = \mathfrak{S}_{\mathsf{SA}}(\{\mathsf{IN}, \mathsf{OUT}\}, \emptyset) = \lambda^0$ for some $\lambda^0 \in \Lambda_{\mathsf{SA}}$. At the statement justification stage, $\Sigma_{\mathsf{SA}}(\varphi) = \{\lambda^0\}$ and $L_{\mathsf{SSJ}}(\varphi) = \mathfrak{S}_{\mathsf{SSJ}}(\{\lambda^0\}, \emptyset)$.

C2. Consider a second case C2 where there is a single argument A with conclusion φ, i.e. $Arg(\{\varphi\}) = \{A\}$, and assume again that $Arg(Cnt(\varphi)) = \emptyset$ and that the outcome of the argument acceptance stage consists of two labellings, i.e. $\mathfrak{L}_{\mathsf{AA}}(\mathcal{AC}) = \{L^1_{\mathsf{AA}}, L^2_{\mathsf{AA}}\}$ such that $L^1_{\mathsf{AA}}(A) = \mathsf{IN}$ while $L^2_{\mathsf{AA}}(A) = \mathsf{OUT}$.

- In the AF approach, at the argument justification stage, as in the case C1, $\Sigma_{\mathsf{AA}}(A) = \{\mathsf{IN}, \mathsf{OUT}\}$ and then $L_{\mathsf{AJ}}(A) = \mathfrak{S}_{\mathsf{AJ}}(\{\mathsf{IN}, \mathsf{OUT}\}) = \lambda$. Hence, at the statement justification stage, we get $\Upsilon_{\mathsf{AJ}}(\varphi) = \{\lambda\}$ and $\Upsilon_{\overline{\mathsf{AJ}}}(\varphi) = \emptyset$ from which $L_{\mathsf{ASJ}}(\varphi) = \mathfrak{S}_{\mathsf{ASJ}}(\{\lambda\}, \emptyset)$ must be the same as in case C1.
- In the SF approach, at the statement acceptance stage, we get $\Upsilon^1_{\mathsf{AA}}(\varphi) = \{\mathsf{IN}\}$ and $\Upsilon^2_{\mathsf{AA}}(\varphi) = \{\mathsf{OUT}\}$. Therefore, $L^1_{\mathsf{SA}}(\varphi) = \mathfrak{S}_{\mathsf{SA}}(\{\mathsf{IN}\}, \emptyset) = \lambda^1$ and $L^2_{\mathsf{SA}}(\varphi) = \mathfrak{S}_{\mathsf{SA}}(\{\mathsf{OUT}\}, \emptyset) = \lambda^2$, for some $\lambda^1, \lambda^2 \in \Lambda_{\mathsf{SA}}$. At the statement justification stage, $\Sigma_{\mathsf{SA}}(\varphi) = \{\lambda^1, \lambda^2\}$, and $L_{\mathsf{SSJ}}(\varphi) = \mathfrak{S}_{\mathsf{SSJ}}(\{\lambda^1, \lambda^2\}, \emptyset)$, which may give rise to a different outcome than in case C1.

We observe that in the cases C1 and C2 the statement justification of φ must be the same by the AF approach, while the statement justification of φ may be different by the SF approach. Hence, we conclude that the AF approach is unable to capture some distinctions which can be captured by the SF approach.

A distinction expressible only by the AF approach.

C3. Consider a case C3, where, similarly to case C1, there are two arguments A and B such that, for some statement φ, $Arg(\{\varphi\}) = \{A, B\}$ and $Arg(Cnt(\varphi)) = \emptyset$. Suppose also that the outcome of the argument acceptance stage consists of two labellings, i.e. $\mathfrak{L}_{\mathsf{AA}}(\mathcal{AC}) = \{L^1_{\mathsf{AA}}, L^2_{\mathsf{AA}}\}$ such that $L^1_{\mathsf{AA}}(A) = \mathsf{IN}$ and $L^1_{\mathsf{AA}}(B) = \mathsf{IN}$, while $L^2_{\mathsf{AA}}(A) = \mathsf{IN}$ and $L^2_{\mathsf{AA}}(B) = \mathsf{OUT}$.

- In the AF approach, at the argument justification stage, we get $\Sigma_{\mathsf{AA}}(A) = \{\mathsf{IN}\}$ and $\Sigma_{\mathsf{AA}}(B) = \{\mathsf{IN}, \mathsf{OUT}\}$. Then let $L_{\mathsf{AJ}}(A) = \mathfrak{S}_{\mathsf{AJ}}(\{\mathsf{IN}\}) = \lambda$ and $L_{\mathsf{AJ}}(B) = \mathfrak{S}_{\mathsf{AJ}}(\{\mathsf{IN}, \mathsf{OUT}\}) = \lambda'$. It follows that $\Upsilon_{\mathsf{AJ}}(\varphi) = \{\lambda, \lambda'\}$ while $\Upsilon_{\overline{\mathsf{AJ}}}(\varphi) = \emptyset$, from which $L_{\mathsf{ASJ}}(\varphi) = \mathfrak{S}_{\mathsf{ASJ}}(\{\lambda, \lambda'\}, \emptyset)$.
- In the SF approach, at the statement acceptance stage, we get $\Upsilon^1_{\mathsf{AA}}(\varphi) = \{\mathsf{IN}\}$, $\Upsilon^2_{\mathsf{AA}}(\varphi) = \{\mathsf{IN}, \mathsf{OUT}\}$, and $L^1_{\mathsf{SA}}(\varphi) = \mathfrak{S}_{\mathsf{SA}}(\{\mathsf{IN}\}, \emptyset) = \lambda^1$, and $L^2_{\mathsf{SA}}(\varphi) = \mathfrak{S}_{\mathsf{SA}}(\{\mathsf{IN}, \mathsf{OUT}\}, \emptyset) = \lambda^2$ for some $\lambda^1, \lambda^2 \in \Lambda_{\mathsf{SA}}$. Then $\Sigma_{\mathsf{SA}}(\varphi) = \{\lambda^1, \lambda^2\}$ on which $L_{\mathsf{SSJ}}(\varphi)$ functionally depends.

C4. Consider now a case C4 which differs from case C3 because there is an additional argument labelling L^3_{AA}, namely $\mathfrak{L}_{\mathsf{AA}}(\mathcal{AC}) = \{L^1_{\mathsf{AA}}, L^2_{\mathsf{AA}}, L^3_{\mathsf{AA}}\}$ where L^1_{AA} and L^2_{AA} are as in case C3, while $L^3_{\mathsf{AA}}(A) = \mathsf{OUT}$ and $L^3_{\mathsf{AA}}(B) = \mathsf{IN}$.

- In the AF approach, at the argument justification stage, we get $\Sigma_{\mathsf{AA}}(A) = \Sigma_{\mathsf{AA}}(B) = \{\mathsf{IN}, \mathsf{OUT}\}$. Then $L_{\mathsf{AJ}}(A) = L_{\mathsf{AJ}}(B) = \mathfrak{S}_{\mathsf{AJ}}(\{\mathsf{IN}, \mathsf{OUT}\}) = \lambda'$. It follows that $\Upsilon_{\mathsf{AJ}}(\varphi) = \{\lambda'\}$ and $L_{\mathsf{ASJ}}(\varphi) = \mathfrak{S}_{\mathsf{ASJ}}(\{\lambda'\}, \emptyset)$, which may give rise to a different outcome than in case C3.
- In the SF approach, at the statement acceptance stage, we get $\Upsilon^3_{\mathsf{AA}}(\varphi) = \Upsilon^2_{\mathsf{AA}}(\varphi) = \{\mathsf{IN}, \mathsf{OUT}\}$, hence $L^3_{\mathsf{SA}}(\varphi) = L^2_{\mathsf{SA}}(\varphi) = \lambda^2$. Then also in the case C4 we get $\Sigma_{\mathsf{SA}}(\varphi) = \{\lambda^1, \lambda^2\}$ and hence $L_{\mathsf{SSJ}}(\varphi)$ must be the same as in case C3.

We observe that in the cases C3 and C4 the statement justification of φ may be different by the AF approach, while the statement justification of φ must be the same by the SF approach. Hence, we conclude that the SF approach is unable to capture some distinctions which can be captured by the AF approach.

In summary, the AF and the SF approaches are incomparable in terms of expressiveness.

4 ARGUMENTATION FORMALISMS AS MLSs

In this section we illustrate how to cast argumentation formalisms into MLSs. In particular we will consider $ASPIC^+$, Assumption-Based Argumentation, and Defeasible Logic Programming and examine for each formalism whether it can be seen as an instance of the argument-focused approach, the statement-focused approach, or both. This analysis involves identifying some argument and statement labellings and the relevant synthesis operators corresponding to the original definition of each formalism. We will also show how this reconstruction in terms of MLSs and associated properties can ease the analysis and comparison of argumentation formalisms.

Due to space limitations we will not recall the definitions of these formalisms, which are typically very rich, but we will rather focus on the properties relevant for our development. The reader is referred to the original references for all the details.

Some of the considered formalisms deal with infinite and/or circular arguments. As these kinds of argument would require a specific additional treatment, due to space limitations and in order to focus on the main message of the paper, we will consider only finite and non-circular arguments in the context of each of the reviewed formalisms.

Moreover, we will assume that the monotonic or strict part of the knowledge base is consistent, that is, it does not support the derivation of contrary conclusions.

Also, we will not formally develop Example 1 for each formalism, but will refer directly to the statement justification outcome, assuming that the underlying formalisation is quite immediate in each case. As a brief informal description, we assume that two mutually attacking arguments support the statements S1 and S2, that the argument supporting the statement S3 is defeated by another (stronger) argument supporting the negation of S3 (denoted ¬S3), and that there are no arguments supporting S4, nor its negation.

In the remainder, all proofs are omitted due to space limitations.

4.1 Basic properties of MLSs

MLSs are useful to analyse and compare actual argumentation formalisms on a common ground consisting of abstract general properties. In particular we will consider in this paper the notions of full coverage and contrary-sensitivity.

The first property requires that the relevant functions are total.

Definition 15 (Coverage). *An AF MLS $\mathfrak{L}^A = \langle \mathcal{AC}, \mathfrak{L}_{\mathsf{AA}}, L_{\mathsf{AJ}}, L_{\mathsf{ASJ}} \rangle$ (resp. a SF MLS $\mathfrak{L}^S = \langle \mathcal{AC}, \mathfrak{L}_{\mathsf{AA}}, \mathfrak{L}_{\mathsf{SA}}, L_{\mathsf{SSJ}} \rangle$) is said to provide*

- *a full coverage of argument acceptance if every $L_{\mathsf{AA}} \in \mathfrak{L}_{\mathsf{AA}}$ is total,*
- *a full coverage of argument justification (resp. of statement acceptance) if L_{AJ} (resp. every $L_{\mathsf{SA}} \in \mathfrak{L}_{\mathsf{SA}}$) is total,*
- *a full coverage of statement justification if L_{ASJ} (resp. L_{SSJ}) is total.*

$\mathfrak{L}^A$ (resp. $\mathfrak{L}^S$) provides an exhaustive coverage if it provides all the three levels of full coverage introduced above.

Sensitivity to contrariness concerns statement justification only: the idea is that the justification status of a statement φ is actually somehow affected also by the contraries of φ. Formally this amounts to require that they make some difference in the evaluation.

Definition 16 (Contrary-sensitivity). *Given an AF MLS $\mathfrak{L}^A = \langle \mathcal{AC}, \mathfrak{L}_{\mathsf{AA}}, L_{\mathsf{AJ}}, L_{\mathsf{ASJ}} \rangle$, L_{ASJ} is* contrary-sensitive *iff $\exists \varphi, \psi \in \mathcal{L}$ such that $\Upsilon_{\mathsf{AJ}}(\varphi) = \Upsilon_{\mathsf{AJ}}(\psi)$, $\Upsilon_{\overline{\mathsf{AJ}}}(\varphi) \neq \Upsilon_{\overline{\mathsf{AJ}}}(\psi)$, and $L_{\mathsf{ASJ}}(\varphi) \neq L_{\mathsf{ASJ}}(\psi)$.*

Given a SF MLS $\mathfrak{L}^S = \langle \mathcal{AC}, \mathfrak{L}_{\mathsf{AA}}, \mathfrak{L}_{\mathsf{SA}}, L_{\mathsf{SSJ}} \rangle$, L_{SSJ} is contrary-sensitive *iff $\exists \varphi, \psi \in \mathcal{L}$ such that $\Sigma_{\mathsf{SA}}(\varphi) = \Sigma_{\mathsf{SA}}(\psi)$, $\Sigma_{\overline{\mathsf{SA}}}(\varphi) \neq \Sigma_{\overline{\mathsf{SA}}}(\psi)$, and $L_{\mathsf{SSJ}}(\varphi) \neq L_{\mathsf{SSJ}}(\psi)$.*

We will use the properties of coverage and contrary-sensitivity to analyse argumentation formalisms in the remainder of this section. So, an argumentation formalism $\mathfrak{F}$ can be considered, at a very general level, as a mechanism to produce ACSs and the relevant labellings. The universe of all ACSs possibly produced by a formalism $\mathfrak{F}$ is denoted as $\Omega(\mathfrak{F})$. Concepts concerning MLSs and their components can be applied to argumentation formalisms considering the elements of $\Omega(\mathfrak{F})$. For instance a formalism $\mathfrak{F}$ is said to provide an exhaustive coverage if all of the MLSs associated to every ACS in $\Omega(\mathfrak{F})$ provide an exhaustive coverage.

4.2 *ASPIC*$^+$

ASPIC$^+$ (denoted as A+ for short) is a rule-based argumentation formalism which assumes the existence of a generic language $\mathcal{L}$ equipped with a contrariness relation [12, 10, 11].

A+ arguments may attack each other, and argument acceptance is based on Dung's formalism of argumentation frameworks [7] and its semantics. Accordingly, it is possible to refer to the labelling-based version of Dung's semantics [2], where a set of three argument acceptance labels is adopted, namely $\Lambda_{\mathsf{AA}}^{\mathsf{IOU}} = \{\mathsf{IN}, \mathsf{OUT}, \mathsf{UN}\}$. The $\Lambda_{\mathsf{AA}}^{\mathsf{IOU}}$-based argument acceptance labellings prescribed by the various abstract argumentation semantics proposed in the literature are all total. Note however that *stable* semantics fails to produce any labelling in some cases. In general, the argument acceptance phase for an argument collection $\mathcal{AC}$ in A+ produces the acceptance evaluation $\mathfrak{L}_{\mathsf{AA}}(\mathcal{AC})$ and the relevant projection Σ_{AA} (Definition 7).

Concerning the subsequent stage, A+ focuses on argument justification, hence it belongs to the AF approach, and adopts the traditional notion of skeptical and credulous justification (see Def. 3.1 of [11]) which says that an argument is skeptically justified (denoted SKJ) if it is labelled IN in all labellings prescribed by the adopted semantics, while it is credulously justified (denoted CRJ) if it is labelled IN in some labellings. It is easy to see that, with a small adjustment to keep the two notions disjoint, it fits Definition 8.

Proposition 1. *Given the set of argument justification labels $\Lambda_{\mathsf{AJ}}^{\mathsf{A+}} = \{\mathsf{SKJ}, \mathsf{CRJ}\}$, the argument justification labelling $L_{\mathsf{AJ}}^{\mathsf{A+}}$ prescribed by* A+ *for every $\mathcal{AC} \in \Omega(\mathsf{A+})$ is such that $L_{\mathsf{AJ}}^{\mathsf{A+}}(A) = \mathfrak{S}_{\mathsf{AJ}}^{\mathsf{A+}}(\Sigma_{\mathsf{AA}}(A))$ where the S-operator $\mathfrak{S}_{\mathsf{AJ}}^{\mathsf{A+}}$ from $\Lambda_{\mathsf{AA}}^{\mathsf{IOU}}$ to $\Lambda_{\mathsf{AJ}}^{\mathsf{A+}}$ is defined for $T \in Pow(\Lambda_{\mathsf{AA}}^{\mathsf{IOU}})$ as*

- *$\mathfrak{S}_{\mathsf{AJ}}^{\mathsf{A+}}(T) = \mathsf{SKJ}$ iff $T = \{\mathsf{IN}\}$;*
- *$\mathfrak{S}_{\mathsf{AJ}}^{\mathsf{A+}}(T) = \mathsf{CRJ}$ iff $T \supsetneq \{\mathsf{IN}\}$.*

It can be immediately observed that literally $L_{\mathsf{AJ}}^{\mathsf{A+}}$ does not provide full coverage, since it does not cover the cases where $\mathsf{IN} \notin \Sigma_{\mathsf{AA}}(A)$. This can be explained by the emphasis on acceptance in A+. It is anyway easy to recover a full coverage by defining a third status (let say *not justified*, denoted as NOJ), covering the remaining cases, i.e. letting $\Lambda_{\mathsf{AJ}}^{\mathsf{A+}} = \{\mathsf{SKJ}, \mathsf{CRJ}, \mathsf{NOJ}\}$.

In A+, statements inherit directly the justification status of the "best justified" argument supporting them: a statement is skeptically justified if and only if it is the conclusion of a skeptically justified argument, while it is credulously justified if and only if it is not skeptically justified and it is the conclusion of a credulously justified argument. Again, it can be proved that this fits Definition 10, applying the transfer of Definition 9 to $L_{\mathsf{AJ}}^{\mathsf{A+}}$.

Proposition 2. *Given the set of statement justification labels $\Lambda_{\mathsf{SJ}}^{\mathsf{A+}} = \{\mathsf{skj}, \mathsf{crj}\}$, the statement justification labelling $L_{\mathsf{ASJ}}^{\mathsf{A+}}$ prescribed by* A+ *for every $\mathcal{AC} \in \Omega(\mathsf{A+})$ is such that $L_{\mathsf{ASJ}}^{\mathsf{A+}}(\varphi) = \mathfrak{S}_{\mathsf{ASJ}}^{\mathsf{A+}}(\Upsilon_{\mathsf{AJ}}(\varphi), \Upsilon_{\overline{\mathsf{AJ}}}(\varphi))$ where the double S-operator $\mathfrak{S}_{\mathsf{ASJ}}^{\mathsf{A+}}$ from $\Lambda_{\mathsf{AJ}}^{\mathsf{A+}}$ to $\Lambda_{\mathsf{SJ}}^{\mathsf{A+}}$ is defined for $T, U \in Pow(\Lambda_{\mathsf{SJ}}^{\mathsf{A+}})$ as*

- *$\mathfrak{S}_{\mathsf{ASJ}}^{\mathsf{A+}}(T, U) = \mathsf{skj}$ iff $\mathsf{SKJ} \in T$;*
- *$\mathfrak{S}_{\mathsf{ASJ}}^{\mathsf{A+}}(T, U) = \mathsf{crj}$ iff $\mathsf{CRJ} \in T$ and $\mathsf{SKJ} \notin T$.*

$L_{\mathsf{ASJ}}^{\mathsf{A+}}$ does not provide full coverage since it does not cover the cases where $\Upsilon_{\mathsf{AJ}}(\varphi) \cap \{\mathsf{SKJ}, \mathsf{CRJ}\} = \emptyset$, i.e. the justification status is left undefined for all the various cases where a statement is not supported by any justified argument. Again this can be easily fixed by defining a third statement label (denoted as noj) covering the remaining cases, i.e. letting $\Lambda_{\mathsf{SJ}}^{\mathsf{A+}} = \{\mathsf{skj}, \mathsf{crj}, \mathsf{noj}\}$. Note, anyway that by the properties of the formalism, that we can not discuss in detail due to space limitation, not all cases are possible. For instance, under some well-formedness hypotheses of the set rules, if a statement is skeptically justified its contraries cannot be skeptically justified nor credulously justified.

With the above analysis and in particular with Proposition 1 and 2 we have built an AF MLS for *ASPIC*$^+$. It can be shown (using the same line of reasoning presented in the second part of Section 3) that it is impossible to build a SF MLS which, starting from $\mathfrak{L}_{\mathsf{AA}}(\mathcal{AC})$, produces the same statement justification labelling as *ASPIC*$^+$ in all cases. More precisely, the situation of the arguments A and B in the case C3 presented in Section 3 can be obtained, for instance[4], with a Dung's argumentation framework consisting of three arguments, A, B, C, where B and C mutually attack each other. Applying stable semantics to this framework gives rise to two labellings L_{AA}^1 and L_{AA}^2 whose restriction on A and B is as described in Section 3. Similarly,

[4] We omit the underlying rule based-reasoning and admit that these small ad-hoc examples can be felt as somehow unrealistic: the same situation for A and B could be obtained in a realistic rule-based reasoning scenario with a larger number of arguments.

the situation of the arguments A and B in the case C4 can be obtained with a Dung's argumentation framework consisting of four arguments, A, B, C, D where A and D mutually attack each other, B and C mutually attack each other, and in addition D attacks C. Again, applying stable semantics to this framework gives rise to three labellings L^1_{AA}, L^2_{AA},L^3_{AA} whose restriction on A and B is as described in Section 3. Under the assumption that $Arg(\{\varphi\}) = \{A, B\}$, we get that in case C3, $L^{\mathsf{A+}}_{\mathsf{AJ}}(A) = \mathsf{SKJ}$, $L^{\mathsf{A+}}_{\mathsf{AJ}}(B) = \mathsf{CRJ}$, from which $L^{\mathsf{A+}}_{\mathsf{SJ}}(\varphi) = \mathsf{skj}$. In case C4 we get $L^{\mathsf{A+}}_{\mathsf{AJ}}(A) = L^{\mathsf{A+}}_{\mathsf{AJ}}(B) = \mathsf{CRJ}$ from which $L^{\mathsf{A+}}_{\mathsf{ASJ}}(\varphi) = \mathsf{crj}$. So the justification of the statement φ is different in the two cases, while as shown in Section 3 this difference can not be obtained in the statement focused model. This shows that the argument and statement justification mechanisms adopted in *ASPIC*$^+$, as, defined in [11], belong to the AF camp. Since *ASPIC*$^+$ is a generic formalism admitting many instances, note also that this does not show that it is in general impossible to reconstruct actual instances of *ASPIC*$^+$ in the SF approach: there can be some instance-specific constraints preventing cases like the ones illustrated above to actually occur.

Moreover it is evident that A+ is not contrary-sensitive, given that $\mathfrak{S}^{\mathsf{A+}}_{\mathsf{ASJ}}$ does not actually use its second parameter U, i.e. $\Upsilon_{\overline{\mathsf{AJ}}}(\varphi)$ does not have any effect in the definition of $L^{\mathsf{A+}}_{\mathsf{SJ}}(\varphi)$. As above, this can be explained by the focus on positive support in A+. Hence these limitations are certainly not intrinsic to A+, rather they can be overcome by providing more articulated definitions for the notions of justification, leaving unchanged all the rest of the formalism.

Example 2. *Referring to Example 1, we observe that according to* A+*, with every semantics, S3 and S4 get the same justification status (undefined or* noj*), while* $\neg S3$ *would be* skj*. The status of S1 and S2 is semantics-dependent: both would get the status* crj *if a Dung multiple-status semantics (e.g. preferred or stable) is adopted, while they would be equated to S3 and S4 (undefined or* noj*) in the case of a Dung single-status semantics (e.g. grounded or ideal)*[5].

4.3 Assumption-Based Argumentation

Assumption-Based Argumentation (denoted as ABA for short) is a rule-based argumentation formalism which, similarly to *ASPIC*$^+$, assumes the existence of a generic language $\mathcal{L}$ equipped with a contrariness relation, see [13] for a tutorial. Similarly to *ASPIC*$^+$, ABA uses Dung's argumentation frameworks and their semantics hence the set of argument acceptance labels $\Lambda^{\mathsf{IOU}}_{\mathsf{AA}} = \{\mathsf{IN}, \mathsf{OUT}, \mathsf{UN}\}$ and the relevant considerations are the same as for *ASPIC*$^+$.

The situation is more articulated concerning the other stages since both a credulous and a skeptical[6] stance [6] have been considered in the literature. In the credulous stance (see a detailed description in [13]) a statement φ is justified if there is at least one acceptance labelling supporting φ. This can be directly reconstructed in the AF approach: using the terminology of [13], an argument is justified if it belongs to at least one *winning* set. This corresponds to being labelled IN in at least one labelling in Definition 8.

Proposition 3. *Given the set of argument justification labels* $\Lambda^{\mathsf{AB\cdot cr}}_{\mathsf{AJ}} = \{\mathsf{WIN}\}$*, the credulous argument justification labelling* $L^{\mathsf{AB\cdot cr}}_{\mathsf{AJ}}$ *prescribed by* ABA *for every* $\mathcal{AC} \in \Omega(ABA)$ *is such that* $L^{\mathsf{AB\cdot cr}}_{\mathsf{AJ}}(A) = \mathfrak{S}^{\mathsf{AB\cdot cr}}_{\mathsf{AJ}}(\Sigma_{\mathsf{AA}}(A))$ *where the S-operator* $\mathfrak{S}^{\mathsf{AB\cdot cr}}_{\mathsf{AJ}}$ *from* $\Lambda^{\mathsf{IOU}}_{\mathsf{AA}}$ *to* $\Lambda^{\mathsf{AB\cdot cr}}_{\mathsf{AJ}}$ *is defined for* $T \in Pow(\Lambda^{\mathsf{IOU}}_{\mathsf{AA}})$ *as* $\mathfrak{S}^{\mathsf{AB\cdot cr}}_{\mathsf{AJ}}(T) = \mathsf{WIN}$ *iff* $T \supseteq \{\mathsf{IN}\}$.

This labelling does not allow full coverage (since it does not cover the cases where $\mathsf{IN} \notin T$), but it is immediate to recover full coverage by introducing a complementary "not winning" label NOWIN.

In the credulous stance of ABA, statements inherit directly the justification status from arguments: a statement is winning if it is the conclusion of a winning argument. This easily fits Definition 10.

Proposition 4. *Given the set of statement justification labels* $\Lambda^{\mathsf{AB\cdot cr}}_{\mathsf{SJ}} = \{\mathsf{win}\}$*, the statement justification labelling* $L^{\mathsf{AB\cdot cr}}_{\mathsf{ASJ}}$ *prescribed by* ABA *for every* $\mathcal{AC} \in \Omega(ABA)$ *is such that* $L^{\mathsf{AB\cdot cr}}_{\mathsf{ASJ}}(\varphi) = \mathfrak{S}^{\mathsf{AB\cdot cr}}_{\mathsf{ASJ}}(\Upsilon_{\mathsf{AJ}}(\varphi), \Upsilon_{\overline{\mathsf{AJ}}}(\varphi))$ *where the double S-operator* $\mathfrak{S}^{\mathsf{AB\cdot cr}}_{\mathsf{ASJ}}$ *from* $\Lambda^{\mathsf{AB\cdot cr}}_{\mathsf{AJ}}$ *to* $\Lambda^{\mathsf{AB\cdot cr}}_{\mathsf{SJ}}$ *is defined for* $T, U \in Pow(\Lambda^{\mathsf{AB\cdot cr}}_{\mathsf{AJ}})$ *as* $\mathfrak{S}^{\mathsf{AB\cdot cr}}_{\mathsf{ASJ}}(T, U) = \mathsf{win}$ *iff* $\mathsf{WIN} \in T$.

Again, literally speaking, this labelling does not provide full coverage (which can however be easily recovered with an additional label nowin) and it is not contrary-sensitive. Here similar observations as for the case of *ASPIC*$^+$ apply.

It can be easily shown that the credulous stance can also be reconstructed in the SF model. We omit it due to space limitation.

Example 3. *According to the credulous stance in* ABA*, with every semantics, S3 and S4 get the same justification status (undefined or* nowin*), while* $\neg S3$ *is* win*. The status of S1 and S2 is semantics-dependent: both get the status* win *(thus being equated to* $\neg S3$*) if a multiple-status semantics is adopted, while they are equated to S3 and S4 (undefined or* nowin*) in the case of a single-status semantics.*

In [6], in addition to the credulous statement justification reviewed above, a skeptical notion of justification is considered: basically a statement φ is skeptically justified if all acceptance labellings support φ. On this basis, a more articulated classification of statement justification, distinguishing credulously, skeptically and not justified statements can be introduced. This stance, denoted as $\mathsf{AB} \cdot \mathsf{sk}$ can be reconstructed in the SF approach as we illustrate below.

Definition 17. *Given the set of statement acceptance labels* $\Lambda^{\mathsf{AB\cdot sk}}_{\mathsf{SA}} = \{\mathsf{in}, \mathsf{nin}\}$*, the double S-operator* $\mathfrak{S}^{\mathsf{AB\cdot sk}}_{\mathsf{SA}}$ *from* $\Lambda^{\mathsf{IOU}}_{\mathsf{AA}}$ *to* $\Lambda^{\mathsf{AB\cdot sk}}_{\mathsf{SA}}$ *for* AB·sk *is defined for* $T, U \in Pow(\Lambda^{\mathsf{IOU}}_{\mathsf{AA}})$ *as:*
- $\mathfrak{S}^{\mathsf{AB\cdot sk}}_{\mathsf{SA}}(T, U) = \mathsf{in}$ *iff* $\mathsf{IN} \in T$*;*
- $\mathfrak{S}^{\mathsf{AB\cdot sk}}_{\mathsf{SA}}(T, U) = \mathsf{nin}$ *otherwise.*

So, by applying $\mathfrak{S}^{\mathsf{AB\cdot sk}}_{\mathsf{SA}}$, for a given statement φ and a single acceptance labelling L_{AA}, if at least one argument supporting φ is labelled IN in L_{AA}, then $L_{\mathsf{SA}}(\varphi) = \mathsf{in}$, else $L_{\mathsf{SA}}(\varphi) = \mathsf{nin}$.

The final stage of statement justification requires the statement justification labels corresponding to skeptical, credulous and no justification, and a double S-operator, which is rather simple since, as the other cases considered above, $\mathsf{AB} \cdot \mathsf{sk}$ is contrary insensitive.

Proposition 5. *Given the set of statement justification labels* $\Lambda^{\mathsf{AB\cdot sk}}_{\mathsf{SJ}} = \{\mathsf{skj}, \mathsf{crj}, \mathsf{noj}\}$*, the statement justification labelling* $L^{\mathsf{AB\cdot sk}}_{\mathsf{SSJ}}$ *prescribed by* ABA *for every* $\mathcal{AC} \in \Omega(ABA)$ *is such that* $L^{\mathsf{AB\cdot sk}}_{\mathsf{SSJ}}(\varphi) = \mathfrak{S}^{\mathsf{AB\cdot sk}}_{\mathsf{SSJ}}(\Sigma_{\mathsf{SA}}(\varphi), \Sigma_{\overline{\mathsf{SA}}}(\varphi))$ *where the double S-operator* $\mathfrak{S}^{\mathsf{AB\cdot sk}}_{\mathsf{SSJ}}$ *from* $\Lambda^{\mathsf{AB\cdot sk}}_{\mathsf{SA}}$ *to* $\Lambda^{\mathsf{AB\cdot sk}}_{\mathsf{SJ}}$ *is defined for* $T, U \in Pow(\Lambda^{\mathsf{AB\cdot sk}}_{\mathsf{AJ}})$ *as*
- $\mathfrak{S}^{\mathsf{AB\cdot sk}}_{\mathsf{SSJ}}(T, U) = \mathsf{skj}$ *iff* $T = \{\mathsf{in}\}$*;*
- $\mathfrak{S}^{\mathsf{AB\cdot sk}}_{\mathsf{SSJ}}(T, U) = \mathsf{crj}$ *iff* $T \supsetneq \{\mathsf{in}\}$*;*
- $\mathfrak{S}^{\mathsf{AB\cdot sk}}_{\mathsf{SSJ}}(T, U) = \mathsf{noj}$ *otherwise.*

[5] An argumentation semantics is *single-status* if, $\forall \mathcal{AC}$ $|\mathfrak{L}_{\mathsf{AA}}(\mathcal{AC})| = 1$, is *multiple-status* if $\exists \mathcal{AC}$ such that $|\mathfrak{L}_{\mathsf{AA}}(\mathcal{AC})| > 1$.

[6] Unfortunately these terms are overloaded in the literature. In particular the skeptical stance of [6] is significantly different e.g. from the notion of skeptical justification in *ASPIC*$^+$. To avoid to introduce a new terminology we are forced to the use of the same term with different meanings in different formalisms.

Proposition 5 completes the reconstruction of $\mathsf{AB} \cdot \mathsf{sk}$ in the SF approach. It is interesting to note that such a reconstruction is not possible in the AF approach: this can be proved with the same line of reasoning used in the first part of Section 3.

Example 4. $\mathsf{AB} \cdot \mathsf{sk}$ *behaves similarly to* $\mathsf{A+}$*: with every semantics, S3 and S4 get the same justification status (undefined or* noj*), while* $\neg$*S3 would be* skj. *The status of S1 and S2 is semantics-dependent: both would get the status* crj *if a Dung multiple-status semantics (e.g. preferred or stable) is adopted, while they would be equated to S3 and S4 (undefined or* noj*) in the case of a Dung single-status semantics.*

4.4 Defeasible Logic Programming

Defeasible Logic Programming (denoted as $DeLP$) "provides a computational reasoning system that uses an argumentation engine to obtain answers from a knowledge base represented using a logic programming language extended with defeasible rules" [8]. $DeLP$ encompasses two forms of contrariness, namely strong and default negation. On this basis, defeat relations between arguments can be defined. For language statements, a notion of complement is introduced, based on strong negation only.

Differently from the approaches surveyed in the previous sections, $DeLP$ does not use Dung's framework for argument acceptance evaluation, rather it adopts a dialectical procedure. This leads to a single status approach where each argument is marked as Defeated or Undefeated, hence the set of argument acceptance labels is defined as $\Lambda^{\mathsf{De}}_{\mathsf{AA}} = \{\mathsf{D}, \mathsf{U}\}$. In other words the dialectical procedure is guaranteed to produce a unique total Λ_{AA}-based acceptance labelling: for every $\mathcal{AC} \in \Omega(DeLP)$ $|\mathfrak{L}_{\mathsf{AA}}(\mathcal{AC})| = 1$. Based on this property, in the AF approach acceptance projection (Definition 7) and argument justification (Definition 8) can be defined essentially as identity.

Proposition 6. *Given the set of argument justification labels* $\Lambda^{\mathsf{De}}_{\mathsf{AJ}} = \{\mathsf{D}, \mathsf{U}\}$*, the argument justification labelling* $L^{\mathsf{De}}_{\mathsf{AJ}}$ *prescribed by* $DeLP$ *for every* $\mathcal{AC} \in \Omega(DeLP)$ *is such that* $L^{\mathsf{De}}_{\mathsf{AJ}}(A) = \mathfrak{S}^{\mathsf{De}}_{\mathsf{AJ}}(\Sigma_{\mathsf{AA}}(A))$ *where the S-operator* $\mathfrak{S}^{\mathsf{De}}_{\mathsf{AJ}}$ *from* $\Lambda^{\mathsf{De}}_{\mathsf{AA}}$ *to* $\Lambda^{\mathsf{De}}_{\mathsf{AJ}}$ *is defined for* $T \in Pow(\Lambda^{\mathsf{De}}_{\mathsf{AA}})$ *as* $\mathfrak{S}^{\mathsf{De}}_{\mathsf{AJ}}(T) = \mathsf{D}$ *iff* $T = \{\mathsf{D}\}$*;* $\mathfrak{S}^{\mathsf{De}}_{\mathsf{AJ}}(T) = \mathsf{U}$ *iff* $T = \{\mathsf{U}\}$.

A statement is said to be warranted if it is the conclusion of an argument whose justification label is U. On this simple basis, an articulated notion of justification status for a statement φ based on four labels (corresponding to the possible answers to a $DeLP$ query) is introduced: yes if φ is warranted; no if the complement of φ is warranted; und(ecided) if neither φ nor its complement are warranted; unk(nown) if φ in not in the signature of the program. This fits Definition 10 as shown by Proposition 7 (see also Table 1).

Proposition 7. *Given the set of statement justification labels* $\Lambda^{\mathsf{De}}_{\mathsf{SJ}} = \{\mathsf{yes}, \mathsf{no}, \mathsf{und}, \mathsf{unk}\}$*, for every* $\mathcal{AC} \in \Omega(DeLP)$*, the statement justification labelling* $L^{\mathsf{De}}_{\mathsf{SJ}}$ *prescribed by* $DeLP$ *is such that* $L^{\mathsf{De}}_{\mathsf{ASJ}}(\varphi) = \mathfrak{S}^{\mathsf{De}}_{\mathsf{ASJ}}(\Upsilon_{\mathsf{AJ}}(\varphi), \Upsilon_{\overline{\mathsf{AJ}}}(\varphi))$ *where the double S-operator* $\mathfrak{S}^{\mathsf{De}}_{\mathsf{ASJ}}$ *from* $\Lambda^{\mathsf{De}}_{\mathsf{AJ}}$ *to* $\Lambda^{\mathsf{De}}_{\mathsf{SJ}}$ *is defined for* $T, U \in Pow(\Lambda^{\mathsf{De}}_{\mathsf{SJ}})$ *as*

- $\mathfrak{S}^{\mathsf{De}}_{\mathsf{ASJ}}(T, U) = \mathsf{yes}$ *iff* $\mathsf{U} \in T$*;*
- $\mathfrak{S}^{\mathsf{De}}_{\mathsf{ASJ}}(T, U) = \mathsf{no}$ *iff* $\mathsf{U} \in U$*;*
- $\mathfrak{S}^{\mathsf{De}}_{\mathsf{ASJ}}(T, U) = \mathsf{und}$ *iff* $T \cup U = \{\mathsf{D}\}$*;*
- $\mathfrak{S}^{\mathsf{De}}_{\mathsf{ASJ}}(T, U) = \mathsf{unk}$ *iff* $T \cup U = \emptyset$.

In contrast with the previous formalisms, this labelling provides a full coverage, since all possible cases for a statement are considered (note in particular that it is guaranteed that $\mathsf{U} \notin \Upsilon_{\mathsf{AJ}}(\varphi) \cap \Upsilon_{\overline{\mathsf{AJ}}}(\varphi)$, i.e. a statement and its complement cannot be both warranted). In particular it distinguishes three cases of non acceptance (while non acceptance was overlooked in the previously surveyed formalisms). This is also due to the fact that this approach is contrary sensitive.

$\Upsilon_{\overline{\mathsf{AJ}}}(\varphi) \setminus \Upsilon_{\mathsf{AJ}}(\varphi)$	$\emptyset$	$\{\mathsf{U}\}$	$\{\mathsf{D}\}$	$\{\mathsf{U}, \mathsf{D}\}$
$\emptyset$	unk	yes	und	yes
$\{\mathsf{U}\}$	no	n.a.	no	n.a.
$\{\mathsf{D}\}$	und	yes	und	yes
$\{\mathsf{U}, \mathsf{D}\}$	no	n.a.	no	n.a.

Table 1. Justification status of statement φ depending on $\Upsilon_{\mathsf{AJ}}(\varphi)$ and $\Upsilon_{\overline{\mathsf{AJ}}}(\varphi)$ in $DeLP$.

Example 5. $DeLP$ *would label S3 as* no*,* $\neg$*S3 as* yes*, both S1 and S2 as* und*, and S4 as* unk.

Since $DeLP$ is single-status at the stage of argument acceptance, it encompasses a single notion of positive justification, while, for instance, *ASPIC*$^+$ distinguishes credulous and skeptical justification. This suggests that combining the most expressive aspects of different approaches may give rise to a more general treatment of the notion of argument and statement justification. Further, it can be shown that (omitted due to space limitations) the $DeLP$ statement justification labelling can be reconstructed in the SF approach too.

5 TOWARDS TUNABLE JUSTIFICATION

From the analysis in the previous section, it emerges that different argumentation formalisms adopt quite different notions of justification, both concerning arguments and statements, featuring different properties and sometimes failing to satisfy some intuitive requirements like full coverage and contrary-sensitivity. However these differences do not seem to be caused by technical motivations, but rather to depend on arbitrary choices based on the intended use of the notion of justification in the presentation of the formalisms themselves. Moreover, some proposals (ABA in the credulous stance and $DeLP$) fit both the AF and SF approach, while others (ABA in the sceptical stance and *ASPIC*$^+$) fit only one (the SF and AF approach respectively). These observations back up our claim that the notion of justification (and in particular of statement justification) has been somehow neglected in the development of argumentation formalisms, often more focused on the notion of argument acceptance. Moreover they suggest that justification notions, instead of being "hardwired" in the definitions, could better be conceived as tunable components of any argumentation formalism, with a role similar to those played by argumentation semantics in *ASPIC*$^+$and ABA. These formalisms do not stick to a single argumentation semantics, rather they assume that one is chosen among the various available ones (including possibly those to be developed in the future).

We aim now at illustrating how our model can be used to build alternative options for statement justification, by providing an example of a generic approach to statement justification, the so-called ignorance-aware labelling. Due to space limits, we do this for the AF approach only, analogous ideas are applicable to the SF approach too.

The ignorance-aware labelling captures different reasons for which a statement is not justified: this may be because it is falsified in some way, or due to to some lack of knowledge or because the available knowledge carries some undecidedness. To support this distinction, we assume the set of labels $\Lambda^{\mathsf{Ia}}_{\mathsf{SJ}} = \{\mathsf{yes}, \mathsf{fal}, \mathsf{unk}, \mathsf{ni}\}$, where the label yes indicates that the statement is justified, the label fal indicates that the statement is falsified, unk stands for an 'unknown' statement, while ni captures other cases of indecision about the statement.

Assuming accordingly the set of labels $\Lambda^{\mathsf{Ia}}_{\mathsf{SJ}}$ and full coverage as a basic requirement, the adoption of the ignorance-aware labelling in a formalism essentially amounts to specify for each possible pair $(\Upsilon_{\mathsf{AJ}}(\varphi), \Upsilon_{\overline{\mathsf{AJ}}}(\varphi))$ the corresponding yes, fal, unk or ni label, i.e a double S-operator $\mathfrak{S}^{\mathsf{Ia}}_{\mathsf{ASJ}}$ from Λ_{AJ} to $\Lambda^{\mathsf{Ia}}_{\mathsf{SJ}}$, where Λ_{AJ} is the set of justification labels adopted in the formalism. For the sake of conciseness, in the following we will leave implicit the definition of the S-operator $\mathfrak{S}^{\mathsf{Ia}}_{\mathsf{ASJ}}$ and will express directly the dependence of the statement labelling on $\Upsilon_{\mathsf{AJ}}(\varphi)$ and $\Upsilon_{\overline{\mathsf{AJ}}}(\varphi)$.

Let us consider *ASPIC*$^+$ first and assume $\Lambda^{\mathsf{A+}}_{\mathsf{AJ}} = \{\mathsf{SKJ}, \mathsf{CRJ}, \mathsf{NOJ}\}$. A first, skeptically oriented, option corresponds to the idea that a statement is labelled yes if it is supported by a skeptically justified argument. A second, credulously oriented, option labels a statement yes if it is supported by a skeptically or credulously justified argument.

Definition 18. *The skeptical ignorance-aware labelling for* ASPIC$^+$ *is defined as follows:*
- $L^{\mathsf{Ia \cdot A+ \cdot sk}}_{\mathsf{ASJ}}(\varphi) = \mathsf{yes}$ *iff* $\mathsf{SKJ} \in \Upsilon_{\mathsf{AJ}}(\varphi)$*;*
- $L^{\mathsf{Ia \cdot A+ \cdot sk}}_{\mathsf{ASJ}}(\varphi) = \mathsf{fal}$ *iff* $\mathsf{SKJ} \in \Upsilon_{\overline{\mathsf{AJ}}}(\varphi)$[7]*;*
- $L^{\mathsf{Ia \cdot A+ \cdot sk}}_{\mathsf{ASJ}}(\varphi) = \mathsf{unk}$ *iff* $\Upsilon_{\overline{\mathsf{AJ}}}(\varphi) \cup \Upsilon_{\mathsf{AJ}}(\varphi) = \emptyset$*;*
- $L^{\mathsf{Ia \cdot A+ \cdot sk}}_{\mathsf{ASJ}}(\varphi) = \mathsf{ni}$ *otherwise.*

Definition 19. *The credulous ignorance-aware labelling for* ASPIC$^+$ *is defined as follows*
- $L^{\mathsf{Ia \cdot A+ \cdot cr}}_{\mathsf{ASJ}}(\varphi) = \mathsf{yes}$ *iff* $\{\mathsf{SKJ}, \mathsf{CRJ}\} \cap \Upsilon_{\mathsf{AJ}}(\varphi) \neq \emptyset$*;*
- $L^{\mathsf{Ia \cdot A+ \cdot cr}}_{\mathsf{ASJ}}(\varphi) = \mathsf{fal}$ *iff* $\{\mathsf{SKJ}, \mathsf{CRJ}\} \cap \Upsilon_{\mathsf{AJ}}(\varphi) = \emptyset$ *and* $\{\mathsf{SKJ}, \mathsf{CRJ}\} \cap \Upsilon_{\overline{\mathsf{AJ}}}(\varphi) \neq \emptyset$*;*
- $L^{\mathsf{Ia \cdot A+ \cdot cr}}_{\mathsf{ASJ}}(\varphi) = \mathsf{unk}$ *iff* $\Upsilon_{\overline{\mathsf{AJ}}}(\varphi) \cup \Upsilon_{\mathsf{AJ}}(\varphi) = \emptyset$*;*
- $L^{\mathsf{Ia \cdot A+ \cdot cr}}_{\mathsf{ASJ}}(\varphi) = \mathsf{ni}$ *otherwise.*

Example 6. *In the skeptical labelling, S1 and S2 are labelled as* ni*, S3 as* fal*, ¬S3 as* yes*, while in the credulous labelling S1 and S2 are labelled as* yes*, S3 as* fal*, ¬S3 as* yes*. In both the skeptical and credulous version, S4 is labelled* unk*.*

Turning to ABA (in the credulous stance), the same observations as for *ASPIC*$^+$ apply to the ignorance-aware labelling counterpart.

Definition 20. *The skeptical ignorance-aware labelling for ABA is defined as follows*
- $L^{\mathsf{Ia \cdot AB \cdot sk}}_{\mathsf{ASJ}}(\varphi) = \mathsf{yes}$ *iff* $\mathsf{WIN} \in \Upsilon_{\mathsf{AJ}}(\varphi)$ *and* $\mathsf{WIN} \notin \Upsilon_{\overline{\mathsf{AJ}}}(\varphi)$*;*
- $L^{\mathsf{Ia \cdot AB \cdot sk}}_{\mathsf{ASJ}}(\varphi) = \mathsf{fal}$ *iff* $\mathsf{WIN} \notin \Upsilon_{\mathsf{AJ}}(\varphi)$ *and* $\mathsf{WIN} \in \Upsilon_{\overline{\mathsf{AJ}}}(\varphi)$*;*
- $L^{\mathsf{Ia \cdot AB \cdot sk}}_{\mathsf{ASJ}}(\varphi) = \mathsf{unk}$ *iff* $\Upsilon_{\overline{\mathsf{AJ}}}(\varphi) \cup \Upsilon_{\mathsf{AJ}}(\varphi) = \emptyset$*;*
- $L^{\mathsf{Ia \cdot AB \cdot sk}}_{\mathsf{ASJ}}(\varphi) = \mathsf{ni}$ *otherwise.*

Definition 21. *The credulous ignorance-aware labelling for ABA is defined as follows*
- $L^{\mathsf{Ia \cdot AB \cdot cr}}_{\mathsf{ASJ}}(\varphi) = \mathsf{yes}$ *iff* $\mathsf{WIN} \in \Upsilon_{\mathsf{AJ}}(\varphi)$*;*
- $L^{\mathsf{Ia \cdot AB \cdot cr}}_{\mathsf{ASJ}}(\varphi) = \mathsf{fal}$ *iff* $\mathsf{WIN} \notin \Upsilon_{\mathsf{AJ}}(\varphi)$ *and* $\mathsf{WIN} \in \Upsilon_{\overline{\mathsf{AJ}}}(\varphi)$*;*
- $L^{\mathsf{Ia \cdot AB \cdot cr}}_{\mathsf{ASJ}}(\varphi) = \mathsf{unk}$ *iff* $\Upsilon_{\overline{\mathsf{AJ}}}(\varphi) \cup \Upsilon_{\mathsf{AJ}}(\varphi) = \emptyset$*;*
- $L^{\mathsf{Ia \cdot AB \cdot cr}}_{\mathsf{ASJ}}(\varphi) = \mathsf{ni}$ *otherwise.*

Example 7. *In the skeptical labelling for ABA, S1 and S2 are labelled as* ni*, S3 as* fal*, ¬S3 as* yes*, S4 as* unk*, while in the credulous case, S1 and S2 are labelled as* yes*, S3 as* fal*, ¬S3 as* yes*, S4 as* unk*.*

As to $DeLP$, the original labelling reified in Proposition 7 is ignorance-aware, modulo two label names: no for fal, und for ni.

[7] We don't specify the additional condition $\mathsf{SKJ} \notin \Upsilon_{\mathsf{AJ}}(\varphi)$ since, by the properties of *ASPIC*$^+$, it is already implied by $\mathsf{SKJ} \in \Upsilon_{\overline{\mathsf{AJ}}}(\varphi)$.

Definition 22. *The (skeptical) ignorance-aware labelling for $DeLP$ is defined as follows*
- $L^{\mathsf{Ia \cdot De}}_{\mathsf{ASJ}}(\varphi) = \mathsf{yes}$ *iff* $\mathsf{U} \in \Upsilon_{\mathsf{AJ}}(\varphi)$*;*
- $L^{\mathsf{Ia \cdot De}}_{\mathsf{ASJ}}(\varphi) = \mathsf{fal}$ *iff* $\mathsf{U} \in \Upsilon_{\overline{\mathsf{AJ}}}(\varphi)$*;*
- $L^{\mathsf{Ia \cdot De}}_{\mathsf{ASJ}}(\varphi) = \mathsf{unk}$ *iff* $\Upsilon_{\overline{\mathsf{AJ}}}(\varphi) \cup \Upsilon_{\mathsf{AJ}}(\varphi) = \emptyset$*;*
- $L^{\mathsf{Ia \cdot De}}_{\mathsf{ASJ}}(\varphi) = \mathsf{ni}$ *otherwise.*

Example 8. *According to Definition 22, S1 and S2 are labelled as* ni*, S3 as* fal*, ¬S3 as* yes*, S4 as* unk*.*

It can be noted that the credulous and skeptical versions of the ignorance-aware labelling provide (respectively) coincident results for all the formalisms considered, while the outcomes were different (not only formally but also substantially) with the original definitions.

Example 9. *For every statement S in $\{S1, S2, S3, \neg S3, S4, S5\}$:*
- $L^{\mathsf{Ia \cdot A+ \cdot sk}}_{\mathsf{ASJ}}(S) = L^{\mathsf{Ia \cdot AB \cdot sk}}_{\mathsf{ASJ}}(S) = L^{\mathsf{Ia \cdot De}}_{\mathsf{ASJ}}(S)$*, and*
- $L^{\mathsf{Ia \cdot A+ \cdot cr}}_{\mathsf{ASJ}}(S) = L^{\mathsf{Ia \cdot AB \cdot cr}}_{\mathsf{ASJ}}(S)$*.*

6 CONCLUSION

Argument-based reasoning is a complex activity which is based on, but is not limited to, the tasks of argument production and argument acceptance evaluation, on which many current literature formalisms are mainly focused. In particular, treating statement justification as a simple byproduct of the previous reasoning stages tends to hide the conceptual richness of this task too. As we have shown, this gives rise to disagreements and/or losses of sensitivity in some well-known formalisms even in simple common-sense examples.

To overcome these limits, we have proposed the novel notion of multi-labelling system for argument-based reasoning, which restores statement justification as a first-class formalism-independent component of the overall process and promotes the idea that it is tunable, much in the way argumentation semantics is a tunable component in several formalisms. We have shown that at least two approaches, namely the AF and the SF approach, can be considered in this context and that they are incomparable in terms of expressiveness. We have then shown how the process leading from argument acceptance to statement justification in three well-known argumentation formalisms can be regarded as a kind of either AF or SF multi-labelling system. Multi-labelling systems can be tuned, and we illustrated this by 'plugging' a (so-called ignorance-aware) labelling for statement justification into the three considered formalisms, thus achieving agreement in the example used in the paper.

Overall, the paper provides a first foundational contribution towards a deeper study of statement justification in argument-based reasoning and opens the way to several future research directions. In particular, we mention a systematic study of general principles and properties for statement labellings. This would represent a complementary contribution with respect to the literature works on rationality postulates [5, 1] which deal with the collective properties (e.g. consistency and closure) of the conclusions of a set of arguments rather than with the notion of justification of the conclusions themselves. Moreover we will consider the investigation of other pluggable statement justification methods and of their relationships, and the revision of some of the assumptions underlying the approaches considered in this paper. For instance, revising some of these assumptions may allow one to overcome the expressiveness gaps evidenced in Section 3. Finally, a first analysis carried out on Defeasible Logic [9] shows the necessity, in this context, of more articulated approaches taking into account the different types of arguments and attacks present in the underlying formalism.

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