On ASPIC$^+$ and Defeasible Logic

Ho-Pun LAM [1], Guido GOVERNATORI and Régis RIVERET
Data61, CSIRO | NICTA, Australia [2]

Abstract. Dung-like argumentation framework ASPIC$^+$ and Defeasible Logic (DL) are both well-studied rule-based formalisms for defeasible reasoning. We compare the two frameworks and establish a linkage between an instantiation of ASPIC$^+$ and a DL variant, which leads to a better understanding and cross-fertilization – in particular our work sheds light on features such as ambiguity propagating/blocking, team defeat and strict rules for argumentation, while emphasizing the argumentation-theoretic features of DL.

Keywords. ASPIC$^+$, Defeasible Logic, argumentation

1. Introduction

The argumentation framework ASPIC$^+$ and Defeasible Logic (DL) support, from different perspectives, rule-based inferences pertaining to defeasible reasoning.

ASPIC$^+$ [20, 16, 17] originates from a project aiming at integrating and consolidating well-studied approaches to structured argumentation. ASPIC$^+$ develops the instantiation of Dung's abstract framework [8] provided in [1]. to give a general structured account of argumentation that is intermediate in its level of abstraction between concrete logics and the fully abstract level, providing guidance on the structure of arguments, the nature of attacks, and the use of preferences, accommodating at the same time a broad range of instantiating logics and allowing for the study of conditions under which the various desirable properties are satisfied by these instantiations.

DL [18, 3] is a simple, efficient but flexible non-monotonic formalism capable of dealing with many different intuitions of non-monotonic reasoning. DL has a very distinctive feature: the logic was designed to be easily implementable right from the beginning, and has linear complexity [13]; DL is a framework hosting different variants of DL; within this framework DL can be "tuned" in order to obtain a logic with desired properties, such as ambiguity blocking/propagation and team defeat.

Dung [8] presented an abstract argumentation framework, and different works showed that several well-known nonmonotonic reasoning systems are concrete instances of the abstract framework. Although DL can be described informally in terms of arguments, the various variants have been formalized in a proof-theoretic setting in which arguments play no role. For this reason, [11] gave an argumentation semantics for the variants of DL. They showed that Dung's grounded semantics characterizes the ambiguity propagation defeasible logic without team defeat.

[1]Corresponding Author: Ho-Pun Lam E-mail: brian.lam@data61.csiro.au
[2]NICTA is funded by the Australian Government through the Department of Communications and the Australian Research Council through the ICT Centre of Excellence Program.

In this paper, we establish close connections between ASPIC$^+$ and DL variants, and highlight their differences. Such connections are meant to lead to a better understanding of each framework, and cross-fertilization. For example, the interpretation of DL proofs in terms of argument interplays shall lead to a more intuitive understanding of DL proof theory, while discussions on ambiguity blocking/propagation in DL shall suggest possible developments in ASPIC$^+$. Since there are already very flexible and efficient implementations of DL, our research may lead to the implementations of argumentation systems on the basis of DL.

This paper is structured as follows. In the next two sections we outline the key concepts of ASPIC$^+$ and DL. The similarities and differences of the two formalisms will be discussed in Section 4. In Section 5, we propose a mapping between an instantiation of ASPIC$^+$ and DL, followed by the conclusions.

2. ASPIC$^+$

ASPIC$^+$ [20, 16, 17] develops Amgoud et al.'s [1] instantiation of Dung's [8] abstract frameworks with accounts of the structure of arguments, the nature of attack and the use of preferences. In the remainder, we will mostly refer to the version given in [17]. The framework posits an unspecified logical language $\mathcal{L}$, and defines arguments as inference trees formed by applying strict or defeasible inference rules to premises that are well formed formulae (wff) in $\mathcal{L}$. A strict rule means that if one accepts the antecedents, then one must accept the consequent no matter what. A defeasible rule means that if one accepts all antecedents, then one must accept the consequent if there is insufficient reason to reject it.

In order to define attacks in the context of a general language $\mathcal{L}$, one needs an appropriately general notion of conflict (i.e., one that does not commit to specific forms of negation). Thus, some minimal assumptions on $\mathcal{L}$ are made; namely that certain wff are a contrary or contradictory of certain other wff. Apart from this, the framework is still abstract: it applies to any set of strict and defeasible inference rules, and to any logical language with a defined contrary relation.

Definition 1. An *argumentation system* is a triple $AS = (\mathcal{L}, \mathcal{R}, n)$ where: (i) $\mathcal{L}$ is a logical language closed under negation ($\neg$). (ii) $\mathcal{R} = \mathcal{R}_s \cup \mathcal{R}_d$ is a set of strict ($\mathcal{R}_s$) and defeasible ($\mathcal{R}_d$) inference rules of the form $\varphi_1, \ldots, \varphi_n \rightarrow \varphi$ and $\varphi_1, \ldots, \varphi_n \Rightarrow \varphi$ respectively (where φ_i, φ are meta-variables ranging over wff in $\mathcal{L}$), and such that $\mathcal{R}_s \cap \mathcal{R}_d = \emptyset$. (iii) n is a partial function such that $n : \mathcal{R}_d \rightarrow \mathcal{L}$.

Informally, $n(r)$ means that r is applicable. To ease the comparison with DL, we assume in the remainder that $\mathcal{L}$ is a language of propositional literals composed from a set of propositional atoms. Given a literal l, $\sim l$ denotes the complement of l, that is, $\sim l = \neg m$ if $l = m$ and $\sim l = m$ if $l = \neg m$.

In ASPIC$^+$, a knowledge base $\mathcal{K}$ is used to specify the premises from which an argument can be built, which is the union of two disjoint kinds of formulae: the axiom $\mathcal{K}_n$ (which cannot be defeated), and the ordinary premises $\mathcal{K}_p$ (which can be defeated).

Definition 2. An *argumentation theory* is a tuple $AT = (AS, \mathcal{K})$ where AS is an argumentation system and $\mathcal{K}$ is a knowledge base in AS.

On the basis of an argumentation theory, arguments can be built. An argument is basically the chain applications of the inference rules starting with elements from the knowledge base. We give here a more compact variant of the definition given in [17].

Definition 3. An *argument* A on the basis of an argumentation theory with a knowledge base $\mathcal{K}$ and an argumentation system $(\mathcal{L}, \mathcal{R}, n)$ is:

- φ if $\varphi \in \mathcal{K}$, with: $\mathsf{Prem}(A) = \{\varphi\}$; $\mathsf{Conc}(A) = \{\varphi\}$; $\mathsf{Sub}(A) = \{A\}$; $\mathsf{Rules}(A) = \emptyset$; $\mathsf{DefRules}(A) = \emptyset$, $\mathsf{TopRule}(A) =$ undefined.
- $A_1, \ldots, A_n \rightarrow / \Rightarrow \psi$ if $A_1, \ldots, A_n$ are arguments such that there exists a strict/defeasible rule $\mathsf{Conc}(A_1), \ldots, \mathsf{Conc}(A_n) \rightarrow / \Rightarrow \psi$ in $\mathcal{R}_s / \mathcal{R}_d$, with:
  - $*$ $\mathsf{Prem}(A) = \mathsf{Prem}(A_1) \cup \ldots \cup \mathsf{Prem}(A_n)$,
  - $*$ $\mathsf{Conc}(A) = \psi$,
  - $*$ $\mathsf{Sub}(A) = \mathsf{Sub}(A_1) \cup \ldots \cup \mathsf{Sub}(A_n) \cup \{A\}$. Note that $A_1, \ldots, A_n$ are referred to as the proper sub-arguments of A,
  - $*$ $\mathsf{DefRules}(A) = \{r \mid r \in \mathsf{Rules}(A), r \in \mathcal{R}_d\}$
  - $*$ $\mathsf{StRules}(A) = \{r \mid r \in \mathsf{Rules}(A), r \in \mathcal{R}_s\}$
  - $*$ $\mathsf{TopRule}(A) = \mathsf{Conc}(A_1), \ldots, \mathsf{Conc}(A_n) \rightarrow / \Rightarrow \psi$

where Prem returns the set of formula from $\mathcal{K}$ (premises) that used to build A, Conc returns its conclusion, Sub returns all its sub-arguments, DefRules and StRules respectively return the set of defeasible and strict rules in A, and TopRule returns the last inference rule applied in A.

Definition 4. An argument A is *strict* if $\mathsf{DefRules}(A) = \emptyset$ and *defeasible* otherwise; *firm* if $\mathsf{Prem}(A) \subseteq K_n$; *plausible* if $\mathsf{Prem}(A) \nsubseteq \mathcal{K}_n$; *fallible* if A is plausible or defeasible.

ASPIC$^+$ emphasises that (i) attacks should only be targeted at fallible elements of the attacked argument, (ii) a distinction between preference dependent and preference independent attacks, which leads to the following definition for attacks and defeats.

Definition 5. Argument A *attacks* B iff A undercuts, rebuts or undermines B. Argument A *undercuts* argument B (on B') iff $\mathsf{Conc}(A) = \neg n(r)$ for some $B' \in \mathsf{Sub}(B)$ such that B''s top rule r is defeasible. Argument A *rebuts* argument B (on B') iff $\mathsf{Conc}(A) = \sim\varphi$ for some $B' \in \mathsf{Sub}(B)$ of the form $B_1'', \ldots, B_n'' \Rightarrow \varphi$. Argument A *undermines* B (on φ) iff $\mathsf{Conc}(A) = \sim\varphi$ for an ordinary premise φ of B.

Note that an attack originating from an argument A requires that its conclusion $\mathsf{Conc}(A)$ is in conflict with some fallible elements – i.e., non-axiom premises, or defeasible rules or conclusions of defeasible rules – in the attacked argument.

The attack relation tells which arguments are in conflict with each other: if two arguments are in conflict then they cannot both be justified. In ASPIC$^+$, it is assumed that an argument A can be used as a counter-argument to B, if A successfully attacks, i.e., defeats, B. Whether an attack from A to B (on its sub-argument B') succeeds as a defeat, may depend on the relative strength of A and B', i.e., whether B' is strictly stronger than, or strictly preferred to A. So, the preferences amongst arguments are specified by a binary ordering $\preceq$ over arguments[3].

Notice that while several methods to assign preference orderings have been proposed in ASPIC$^+$, ASPIC$^+$ as a framework does not make any assumption on the argument ordering. To facilitate the comparison with DL we consider the following "last-link" inspired ordering:

- from amongst all the defeasible rules in B there exists a rule which is weaker than (strictly less than according to $\preceq$) all the last defeasible rules in A, and

[3] As is usual, its strict counterpart $\prec$ is defined as $X \prec Y$ iff $X \preceq Y$ and $Y \npreceq X$.

- from amongst all the ordinary premises in B there is an ordinary premise which is weaker than (strictly less than according to $\preceq'$) all the last ordinary premises in A.

On the basis of the preferences over arguments, successful attacks (defeats) are defined.

Definition 6. Let A and B be arguments. *A successfully rebuts B* if A rebuts B on B' and $A \not\prec B'$. *A successfully undermines B* if A undermines B on φ and $A \not\prec \varphi$. *A defeats B* iff A undercuts or successfully rebuts or successfully undermines B.

Let us recap. Based on an argumentation theory (see Def. 2), we can build arguments (Def. 3), attack (Def. 5) and defeat relations (Def. 6), and finally a Dung's argumentation framework [8] can be built. ASPIC$^+$ addresses such constructions by considering the concept of structured argumentation framework.

Definition 7. Let AT be an argumentation theory (AS, KB). A *structured argumentation framework* (SAF) defined by AT, is a triple $(\mathcal{A}, C, \preceq)$ where $\mathcal{A}$ is the smallest set of all finite arguments constructed from KB in AS satisfying Def. 3; $\preceq$ is an ordering on $\mathcal{A}$; $(X, Y) \in C$ iff X attacks Y.

Notice that a structured argumentation framework is defined with respect to finite arguments, and thus infinite arguments are ignored. Eventually, a Dung framework can then be instantiated with ASPIC$^+$ arguments and the ASPIC$^+$ defeat relation.

Definition 8. An *abstract argumentation framework* corresponding to a SAF = $(\mathcal{A}, C, \preceq)$ is a pair (A, D) such that D is the defeat relation on $\mathcal{A}$ determined by $(\mathcal{A}, C, \preceq)$.

From this argumentation graph, the justified arguments can be computed, using standard definition of arguments, acceptable arguments and extensions in Dung's abstract argumentation semantics [8]. In this paper, we will focus on the grounded extension. A conclusion φ is *justified* if, and only if, at least one argument, whose conclusion φ is in the grounded extension.

3. Defeasible Logic (DL)

Knowledge in DL is a triple $(F, R, >)$ where F is a finite set of facts, R is a finite set of rules and $>$ is a binary relation on R called superiority relation. In expressing the proof we consider only propositional rules. Rules containing free variables are interpreted as the set of their variable-free instances. There are three types of rules: (i) *Strict rules*, (ii) *Defeasible rules*, and (iii) *Defeaters*. The definition of *Strict rules* and *defeasible rules* in DL are essentially the same as in ASPIC$^+$; *defeaters* are a special kind of rules that can only prevent some conclusions, but not actively support them. For example, the rule $heavy(X) \rightsquigarrow \neg flies(X)$ states that an animal being heavy is not sufficient enough to conclude that it does not fly. It is only evidence against the conclusion that a heavy animal flies.

A *superiority relation* on R is a relation $>$ on R. Where $r_1 > r_2$, then r_1 is called superior to r_2 and r_2 is inferior to r_1, which express that r_1 may override r_2. The superiority relation indicates the relative strength of two rules. For example, given the defeasible rules:

$$r_1 : bird(X) \Rightarrow flies(X) \qquad r_2 : brokenWing(X) \Rightarrow \neg flies(X)$$

which contradict one another. If the superiority relation is empty we are not able to determine which of the two rules prevails over the other. Hence, no conclusive decision can be made about whether a bird with broken wings can fly, But if we introduce a superiority relation $>$ with $r_2 > r_1$, with the intended meaning that r_2 is strictly stronger than r_1, then we can indeed conclude that the bird cannot fly.

In the following, we use $A(r)$ to denote the set of literals that appears in the body of the rule r, R_s (respectively, R_d) denotes the set of strict (defeasible) rules in R, and $R[q]$ denotes the set of rules with head q.

We now give a short informal presentation of how conclusions are drawn in DL. Let D be a theory in DL (described above). A *conclusion* of D is a tagged literal and can have one of the following four forms: (i) $+\Delta l$ meaning that we have a definite derivation of l; (ii) $-\Delta l$ meaning that we do not have a definite derivation of l; (iii) $+\partial l$ meaning that we have a defeasible derivation of l; (iv) $-\partial l$ meaning that we do not have a defeasible derivation of l.

Provability is defined below grounded on the concept of a derivation (or proof, which is a finite sequence of tagged literals) in a DL theory D. Given a proof P we use $P(n)$ to denote the n-th element of the sequence, and $P(1..n)$ denotes the first n elements of P. Given a DL theory D and a proof tag #, #(D) denotes the set of literals provable with tag # in D.

Definition 9. Given a DL theory D and a proof tag $\# \in \{\Delta, \partial\}$, we have the following:

- A rule $r \in R$ is #-applicable at $n + 1$ if, and only if, $\forall l \in A(r)$, $+\#l \in P(1..n)$.
- A rule $r \in R$ is #-discarded at $n + 1$ if, and only if, $\exists l \in A(r)$, $-\#l \in P(1..n)$.

The definition above means that a rule is applicable (at $n + 1$) if all its antecedent are provable; or discarded if at least one of the literals in the antecedent has been rejected in the derivation.

Strict (or definite) derivations are obtained by forward chaining of facts and strict rules while a defeasible conclusion q can be derived if there is a rule whose conclusion is q, whose prerequisites (antecedent) have either already been proved or given in the case at hand (i.e. facts), and any stronger rule whose conclusion is $\sim q$ has prerequisites that fail to be derived. In other words, a conclusion q is (defeasibly) derivable when:

$+\partial$) If $P(n + 1) = +\partial q$ then either
  (1) $+\Delta q \in P(1..n)$; or
  (2) $-\Delta{\sim}q \in P(1..n)$, and
    (1) $\exists r \in R_{sd}[q]$ such that r is ∂-applicable, and
    (2) $\forall s \in R[\sim q]$ either
      (1) s is ∂-discarded; or
      (2) $\exists t \in R_{sd}[q]$ such that t is ∂-applicable and $t > s$.

The inference conditions for negative proof tags ($-\Delta$ and $-\partial$) are derived from the inference conditions for the corresponding positive proof tags by applying the Principle of Strong Negation introduced by [2]: the strong negation of a formula is closely related to the function that simplifies it by moving all negations to an innermost position in the resulting formula and replace the positive tags with the respective negative tags and vice-versa.

3.1. Ambiguity Blocking and Ambiguity Propagation

A conclusion is ambiguous if there are arguments for it and arguments for its opposite and there are no ways to solve the conflict. Consider, for instance, the following example.

Example 1 (Presumption of Innocence)**.** Consider a DL theory with the following rules.

$$
\begin{array}{llll}
r_1: & evidenceA \Rightarrow \neg responsible & & \\
r_2: & evidenceB \Rightarrow responsible & r_3: & responsible \Rightarrow guilty \\
 & & r_4: & \Rightarrow \neg guilty
\end{array}
$$

and there is no additional information to determine the strength of the rules (i.e., in DL the superiority relation is empty, and there are no preferences on the rules in ASPIC$^+$). Given both *evidenceA* and *evidenceB*, the literal *responsible* is ambiguous since the rules r_1 and r_2, each supporting the negation of the other, are applicable and of the same strength. As a consequence, r_3 is not an applicable rule supporting the *guilty* verdict. We refer to this behaviour as *ambiguity blocking* since the support of *guilty* is blocked by *responsible*, which is the default semantics of DL. Accordingly, we obtain $+\partial\neg guilty$.

Notice that there are no justified arguments in the grounded extension, thus $\neg guilty$ is not a skeptical conclusion in ASPIC$^+$.

However, in some cases, it may be preferable for ambiguity to be propagated from *responsible* to *guilty* since we are reserving the judgment of whether the literal *responsible* is provable or not, but possibly it could be. Consequently the literals *guilty* and $\neg guilty$ are ambiguous; hence an undisputed conclusions cannot be drawn, and we refer to this behaviour as *ambiguity propagation*. Considering the example above, is it appropriate to say that we have reached a not guilty verdict without any reasonable doubt given the fact that the defendant was responsible has not been refuted?[4]

The ambiguity propagation variant of DL, for which we use δ as defeasible proof tag, can be easily achieved by making minor changes to the inference conditions for $+\partial$, as shown below [2].

$+\delta$) If $P(n+1) = +\delta q$ then either
  (1) $+\Delta q \in P(1..n)$; or
  (2) $-\Delta{\sim}q \in P(1..n)$, and
    (1) $\exists r \in R_{sd}[q]$, r is δ-applicable, and
    (2) $\forall s \in R[{\sim}q]$ either
      (1) s is σ-discarded; or
      (2) $\exists t \in R_{sd}[q]$ such that
        t is δ-applicable and $t > s$.

$+\sigma$) If $P(n+1) = +\sigma q$ then either
  (1) $+\Delta q \in P(1..n)$ or
  (2) (1) $-\Delta{\sim}q \in P(1..n)$, and
    (2) $\exists r \in R_{sd}[q]$ such that
      (1) r is σ-applicable, and
      (2) $\forall s \in R[{\sim}q]$ either
        s is δ-discarded or $s \not> r$.

Their explanation is similar to that of $+\partial$. The major difference is that to prove q this time we make it easier to attack it (clause 2.2). Instead of asking that the arguments attacking it are justified arguments, we just ask for defensible arguments, that is rules whose premises are just supported (i.e., there is a valid chain of reasoning leading to it), denoted by $+\sigma$.

Example 1 (continued)**.** Under ambiguity propagation, we obtain $+\sigma guilty$ and $+\sigma\neg guilty$ as they are all supported in the theory. Hence, we obtain $-\delta guilty$ and $-\delta\neg guilty$.

The question of the example above is whether it is appropriate to say that we have reach a not guilty verdict without any reasonable doubt. The evidence supporting that the defendant was responsible has not been refuted.

Example 2. Let us extend the previous example. Suppose that the legal system allows for compensation for wrongly accused people. A person (defendant) has been wrongly accused

[4]For an in-depth discussion of ambiguity propagation and ambiguity blocking in the context of legal reasoning and their relationships with proof standards see [9].

if the defendant is found innocent, where innocent is defined as $\neg guilty$. In addition, by default, people are not entitled to compensation. The additional elements of this scenario are modelled by the following rules:

$$r_5: \neg guilty \Rightarrow innocent \qquad r_6: innocent \Rightarrow compensation$$
$$r_7: \Rightarrow \neg compensation$$

where $r_6 > r_7$.

So, if we adopt ambiguity blocking, then we have that despite there is some doubt about responsibility and, consequently, we cannot rule out that the defendant was wrongly accused, the conclusion is that the defendant is entitled to be compensated for having been wrongly accused. Ambiguity propagation does not allow us to draw the same conclusion; in fact we have $-\delta compensation$.

3.2. Team Defeat

The proof conditions above incorporate the idea of team defeat. That is, an attack on a rule with head l by a rule with head $\neg l$ may be defeated by a different rule with head l.

Example 3. Suppose that a crusader has been given the order by his captain not to kill the enemy, and by his general to kill the enemy. Moreover his priest told that they should not kill the enemy, but the bishop told them to kill the enemy. The theory modelling this scenario contains the rules:

$$r_1: general \Rightarrow kill \qquad r_2: bishop \Rightarrow kill$$
$$r_1': captain \Rightarrow \neg kill \qquad r_2': priest \Rightarrow \neg kill$$

the facts are *general*, *bishop*, *captain* and *priest*; and the superiority relation is $r_1 > r_1'$ and $r_2 > r_2'$. All rules are applicable, so we can argue pro *kill* using r_1, then we have to consider all possible attacks to it. r_1' is defeated by r_1 itself and r_2' is defeated by r_2. So *kill* is justified (i.e., $+\partial kill$) since for every reason against this conclusion there is a stronger reason defeating it (r_1 and r_2 respectively).

Alternatively, we can say that there are two distinct hierarchies of rules both converging to the same conclusion. It is easy to verify that there are no justified arguments concluding *kill* in the grounded extension of the theory when the preference over the rules is the same as the superiority relation in DL, thus *kill* is not a skeptical conclusion in ASPIC$^+$ under the grounded semantics.

Even though the idea of team defeat is natural, it is worth noting that it is not adopted by many related systems and concrete systems of argumentation. On the other hand, the notion of accrual of arguments [19] is gaining more prominence, and team defeat is a form of accrual (albeit one that can only strengthen the arguments in the team).

In case this feature is not desired, DL provides variants of the proof conditions given so far to reject it. The proof conditions for the variants without team defeat can be obtained from the corresponding proof conditions given above with the following changes [6]:

- For $+\partial$ and $+\delta$, clause (2.2.2) is replaced by $r > s$; we use $+\partial^*$ and $+\delta^*$, for the proof tags thus obtained.
- For $+\sigma$ and $-\sigma$, the occurrences of $+\delta$ and $-\delta$ is replaced by $+\delta^*$ and $-\delta^*$, for the proof tags thus obtained we use $+\sigma^*$ and $-\sigma^*$, respectively.

Accordingly, to prove a conclusion we must have an applicable rule which is stronger than all applicable/non discarded rules for the negation of the conclusion we want to prove.

The logical properties of DL have been thoroughly investigated [3, 6]; in particular the relationships between the various proof tags are stated in the following theorem.

Theorem 1 (Inclusion Theorem)**.** *[6, 9] Given a DL theory D, we have:*

- $+\Delta(D) \subseteq +\delta^*(D) \subseteq +\delta(D) \subseteq +\partial(D) \subseteq +\sigma(D) \subseteq +\sigma^*(D)$*;*
- $+\delta^*(D) \subseteq +\partial^*(D) \subseteq +\sigma^*(D)$.

There are theories where all the inclusions are proper.

Notice that the conditions for team defeat are more general than the corresponding conditions where this feature does not hold. Besides the set of conclusions we can derive under ambiguity blocking are, in general, different. However, this is not the case for ambiguity propagation where one set of conclusions is included in the other as we have two chains of proof conditions, and that the set of conclusions we can derive from one proof tags in one chain are different from the set of conclusions we can derive from a proof tag in the other chain.

As DL is skeptical in nature, unless otherwise specified, the discussion below will be focused on skeptical semantics.

4. Acceptability of Arguments: ASPIC$^+$ vs Defeasible Logic

As we have seen in the previous section, while ASPIC$^+$ and DL share many similarities in both the set of features and inference processes, there are several substantial differences. In this section, we are going to describe some of them.

Both formulae are *relative consistent* (or *indirect consistent* in ASPIC$^+$ term [7]). That is, a theory cannot conclude that both a proposition p and its negation are justified unless they are both supported by the monotonic part (strict rules) of the theory [3]. Researchers on both sides do not consider this notion as a weakness of the logics [5, 21]. Instead, [21] believe that this is a strength to ASPIC$^+$ as this makes a wide range of alternative logical instantiations of ASPIC$^+$ possible. However, both researchers agree that undesirable conclusions could be inferred if inconsistency appears in the monotonic part of the theories.

In general, we have two variants of argumentation semantics of DL, namely (i) ambiguity blocking (which corresponds to the semantics of DL), (ii) ambiguity propagation (which corresponds to the grounded semantics of Dung's argumentation framework) [11]. DL is neutral about ambiguity blocking and ambiguity propagation. It is possible to justify both views on ambiguity and that both views have their own sphere of applicability. There are applications where ambiguity blocking is counterintuitive and there are applications where ambiguity propagation is counterintuitive, and there are applications that need both.

The outcome of the discussions here is that a (skeptical) non-monotonic formalism should be able to accommodate both. Through varying the semantics of the proof conditions, DL allows us to use the same language without the need to modify a rule/knowledge base to capture different intuitions under different scenarios. Indeed, several variants [2], including support of well-founded semantics, have been defined to cater for the needs of different situations.

Unlike ASPIC$^+$ that support negation as failure (NAF), DL is an early approach to skeptical non-monotonic reasoning without NAF. That is, DL does not support NAF by default. However, it is possible for us to capture this behavior in DL. For instance, consider the rule below.

$$r:\ B, \mathsf{naf} a \Rightarrow q$$

where B is a set of positive literals.

We can transform the weak negated literal $\mathsf{naf}a$ to not_a and introduce new propositions and rule below to simulate the effect of NAF.

$$\begin{aligned} r &: B, not\_a \Rightarrow q \\ r_a^- &: \ a \Rightarrow \neg not\_a \\ r_a^+ &: \quad \Rightarrow not\_a \\ r_a^- &> r_a^+ \end{aligned}$$

In DL, conclusions with negative proof tags are generated when the literals is rejected by the theory, and no conclusions will be inferred if the literal is *undecidable* [14]. However, in ASPIC+, there is no general notion of rejected conclusion. Even though one could say that a conclusion is credulously/skeptically rejected if one of its contraries is credulously/skeptically accepted, then this notion of rejection would again be based on arguments. Consider the theory containing only the rules below.

$$\begin{aligned} p &\Rightarrow p \\ p &\Rightarrow q \\ &\Rightarrow \neg q \end{aligned}$$

DL cannot infer any conclusions as p is undecidable unless we reason on the theory using *well-founded* semantics [15]. In such case, p will be rejected, subsequently inferring the conclusions $-\delta^* p$, $-\delta^* q$ and $+\delta^* \neg q$. For decisive theories, i.e., theories without undecided literals, the negative extension of a theory (i.e., $\{l : D \vdash -\partial l\}$) is the complement of the positive extension (i.e., $\{l : D \vdash +\partial l\}$). In other terms, if one extends the in/out labelling from arguments to conclusion (see, [4]), then $\mathsf{out}(\mathcal{L}) = \mathcal{L} \setminus \mathsf{in}(\mathcal{L})$, where $\mathsf{in}(\mathcal{L})$ and $\mathsf{out}(\mathcal{L})$ are the set of literals in $\mathcal{L}$ labeled in and out, respectively.

On the other hand, since ASPIC+ does not support infinite arguments, there are no arguments about p and the state of its conclusion is the same as "before" the argumentation process. Then the question is: what is the default state of p that when the argumentation process does not classify as accepted (nor rejected)? It seems that this definition is missing in ASPIC+.

DL contains a feature called *defeater* ($\leadsto$), which can be used to prevent some conclusions from inferred, while ASPIC+ does not. However, this difference is not that significant under (normal) logic programming as we can always transform a DL theory with defeater to an equivalent DL theory without defeater using the transformation described in [3]. However, this may make a difference in Modal Defeasible Logic as defeaters may be used to capture the notion of permission [12].

5. Mapping ASPIC+ to DL

In this section we are going to establish a formal relationship between an instantiation of ASPIC+ and DL. In particular, we assume: (i) the contrariness relation in ASPIC+ is an involutive negation, (ii) the last-link ordering discussed in Section 2, and (iii) and the preference ordering over ordinary premises is empty, i.e., $\leq' = \emptyset$. We prove that ASPIC+ under ground semantics corresponds to the ambiguity propagation no team defeat variant of DL. To begin with, lets consider the example below which shows the differences of the two formalisms.

Example 4. (extracted from [21]) Consider an argument A with a strict top rule for x and an argument B with a defeasible top rule for $\neg x$, as shown below.

$$\begin{aligned} A &: \Rightarrow p, p \Rightarrow q, q \Rightarrow r, r \rightarrow x \\ B &: \rightarrow d, d \rightarrow e, e \rightarrow f, f \Rightarrow \neg x \end{aligned}$$

It can be observed that A asymmetrically attacks B. So, in ASPIC$^+$, x is concluded instead of $\neg x$.

However, the case in DL is a bit different. DL concerns only whether a literal is supported in the inference process, irrespective of the type of rule(s) being used. So, if we infer the above arguments in DL, we have the following conclusions:

$$\begin{array}{ll} D \vdash_{DL} -\Delta x & D \vdash_{DL} -\Delta\neg x \\ D \vdash_{DL} +\sigma^* x & D \vdash_{DL} +\sigma^*\neg x \end{array}$$

That is, both x and $\neg x$ are supported by the DL theory D containing only the rules above (used in arguments A and B) and attack each others with the same strength. Hence, both will be rejected (i.e., $-\delta^* x$ and $-\delta^*\neg x$) in DL. However, if we specify that $r \to x > f \Rightarrow \neg x$, we are able to conclude $+\delta^* x$.

Hence, despite the similarities, it is not possible to use directly an ASPIC$^+$ knowledge base as a DL theory and the other way around. This is due to the treatment of (defeasible) arguments in DL which involve strict rules.

To establish the correspondence between ASPIC$^+$ and DL we introduce a mapping from ASPIC$^+$ theories to DL theories, based on the ambiguity propagation variant of DL without team defeat, as shown in the definition below. We assume that the same propositional language $\mathcal{L}$ has been used in both ASPIC$^+$ and DL.

Definition 10. Let $AT = ((\mathcal{L}, \mathcal{R}, n), \mathcal{K})$ be an ASPIC$^+$ theory and $D = (F, R, >)$ be a DL theory. An argument mapping is a function $D = T(TA)$ that map an argument in AT to rules in DL, such that:

$$\begin{aligned} F &= \mathcal{K}_n \\ R &= \{r :\ \Rightarrow q \mid q \in \mathcal{K}_p\} \cup \mathcal{R} \\ >&= \{r > s \mid (s \leq r) \in \leq\} \cup \\ &\quad\ \{r > s \mid r \in \mathcal{R}_s[q],\ s \in \mathcal{R}_d[\sim q]\} \cup \\ &\quad\ \{r > s \mid r \in \mathcal{R}[\sim q],\ s \in R[q] \text{ such that } q \in \mathcal{K}_p\} \end{aligned}$$

In the transformation, knowledge in $\mathcal{K}$ has been transformed into different features in DL according to their nature. For instance, axioms ($\mathcal{K}_n$) are information that cannot be defeated and will be mapped into facts directly (in DL) without any transformation; while ordinary premises ($\mathcal{K}_p$) are information that can be defeated when arguments with stronger support appear, and are transformed into defeasible rules.

Regarding the preference order, note that besides including all preference order that appears in $\leq$, the transformation includes also the superiority relations between defeasible rules and their conflicting strict rules in $\mathcal{R}$, and those rules that are generated (in the transformed theory) based on the ordinary premises ($\mathcal{K}_p$).[5] The former is used to ensure that the support of literal in the defeasible rule can be blocked (under superiority relation) when applicable conflicting strict rules are appeared during the inference process; whereas the latter is used to defeat ordinary premises with a stronger argument.

We are now prepared to give the relationship between ASPIC$^+$ and DL.

Theorem 2. *Let $AT = ((\mathcal{L}, \mathcal{R}, n), \mathcal{K})$ be an ASPIC$^+$ argumentation theory and $p \in \mathcal{L}$,*

(i) $AT \vdash^{GS}_{A+} p \iff T(AT) \vdash_{DL} +\Delta p$

(ii) $AT \mid\!\sim^{GS}_{A+} p \iff T(AT) \vdash_{DL} +\delta^* p$

[5]Note that the $R[q]$ in the last case of $>$ refers to the rules introduced due to the ordinary premises $\mathcal{K}_p$.

where $AT \vdash^{GS}_{A+} p$ and $AT \mid\!\sim^{GS}_{A+} p$ means that p is *strictly* and *defeasibly* justified in the argumentation theory AT using the grounded semantics in ASPIC$^+$, respectively.

Proof. (sketch) The proof is by induction on the length of a derivation in DL and the number of iterations of the application of the characteristic function $\mathcal{F}_G$ in the construction of the fixed-point of the set of acceptable arguments. The inductive base is straightforward given that the base of acceptability for ASPIC$^+$ is whether a literal is an axiom in $\mathcal{K}_n$ or not, and for DL is being a fact or not. But facts in the DL theory corresponds to the axioms in ASPIC$^+$ argumentation system. For the inductive step we first notice that $+\sigma p$ means that, in ASPIC$^+$, there is an undefeated argument for p, and that the argument is not undercut (all the antecedents are under the inductive hypothesis) and the last step is to see that there are not attacking (undefeated) arguments for $\sim p$. □

As can be seen, an ASPIC$^+$ argumentation system can be transformed into a DL theory by applying the transformations above. It is immediate to see that the mapping from a ASPIC$^+$ argumentation theory to the corresponding DL theory is, in the worse case, *quadratic*, given that we have to consider the relationship between conflicting rules and arguments to derive the superiority relations. Hence, given that the complexity of computing the extensions of DL is linear w.r.t. the size of the theory [13], we have the following result.

Corollary 3. *Acceptability of a proposition in ASPIC$^+$ under grounded semantics can be computed in polynomial time.*

6. Conclusions

In this paper we addressed the question of how to instantiate ASPIC$^+$ in DL. For the other direction, it is possible to capture the ambiguity propagation no team defeat variant of DL in ASPIC$^+$, given that such a variant of DL is characterised by the grounded semantics and, the two formalisms share the same language. Thus a theory in DL is indistinguishable from an argumentation theory in ASPIC$^+$. Moreover, other variants are characterised by skeptical argumentation semantics different from grounded semantics, and, to the best of our knowledge, the relationships between such semantics and ASPIC$^+$ have not been studied.

While it is possible to adopt different argumentation semantics to be applied on top of ASPIC$^+$, this step alone might not be enough to model defeasible logic as an instance of ASPIC$^+$. For example, DL with ambiguity blocking would requires to introduce a second "attack" relation on arguments (see [11]) with a ripple down effects on the ASPIC$^+$ definitions setting the various statuses of the argument. Similarly, DL with team defeat would require changes in the definition of what arguments are: an argument would be a set of proof trees instead of a single proof tree [10]. In this paper, we do not address such issues. However, they show that there is potential for cross-fertilization for research on the relationship between ASPIC$^+$ and DL.

References

[1] L. Amgoud, L. Bodenstaff, M. Caminada, P. McBurney, S. Parsons, H. Prakken, J. van Veenen, and G. Vreeswijk. Final review and report on formal argumentation systems. Technical Report Deliverable D2.6, ASPIC IST-FP6-002307, 2006.

[2] G. Antoniou, D. Billington, G. Governatori, and M. J. Maher. A Flexible Framework for Defeasible Logics. In *Proceedings of the 17th National Conference on Artificial Intelligence*, pages 405–410. AAAI Press / The MIT Press, 2000.

[3] G. Antoniou, D. Billington, G. Governatori, and M. J. Maher. Representation Results for Defeasible Logic. *ACM Transactions on Computational Logic*, 2(2):255–286, 2001.

[4] P. Baroni, G. Governatori, H.-P. Lam, and R. Riveret. On the Justification of Statements in Argumentation-based Reasoning. In J. Delgrande and F. Wolter, editors, *Proc KR 2016*. AAAI Press, Cape Town, South Africa, Apr. 2016.

[5] D. Billington. Defeasible Logic is Stable. *Journal of Logic and Computation*, 3(4): 379–400, 1993.

[6] D. Billington, G. Antoniou, G. Governatori, and M. J. Maher. An Inclusion Theorem for Defeasible Logics. *ACM Transactions on Computational Logic*, 12(1):6:1–6:27, Nov. 2010.

[7] M. Caminada and L. Amgoud. On the evaluation of argumentation formalisms. *Artificial Intelligence*, 171(5-6):286 – 310, 2007.

[8] P. M. Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial Intelligence*, 77 (2):321–358, 1995.

[9] G. Governatori. On the relationship between Carneades and defeasible logic. In *Proc ICAIL 2011*, pages 31–40. ACM, 2011.

[10] G. Governatori and M. J. Maher. An Argumentation-Theoretic Characterization of Defeasible Logic. In W. Horn, editor, *Proc ECAI 2000*, pages 469–474, 2000.

[11] G. Governatori, M. J. Maher, G. Antoniou, and D. Billington. Argumentation Semantics for Defeasible Logic. *Journal of Logic and Computation*, 14(5):675–702, 2004.

[12] G. Governatori, F. Olivieri, A. Rotolo, and S. Scannapieco. Computing Strong and Weak Permissions in Defeasible Logic. *Journal of Philosophical Logic*, 42(6): 799–829, 2013.

[13] M. J. Maher. Propositional Defeasible Logic has Linear Complexity. *Theory and Practice of Logic Programming*, 1(6):691–711, 2001.

[14] M. J. Maher. Relative expressiveness of defeasible logics. *Theory and Practice of Logic Programming*, 12(4-5):793–810, 2012.

[15] M. J. Maher and G. Governatori. A Semantic Decomposition of Defeasible Logics. In *Proc AAAI-99*, pages 299–305, Menlo Park, CA, USA, 1999. AAAI Press.

[16] S. Modgil and H. Prakken. A General Account of Argumentation with Preferences. *Artificial Intelligence*, 195:361–397, Feb. 2013.

[17] S. Modgil and H. Prakken. The ASPIC$^+$ framework for structured argumentation: a tutorial. *Argument & Computation*, 5:31–62, 2014.

[18] D. Nute. Defeasible Logic. In D. Gabbay and C. Hogger, editors, *Handbook of Logic for Artificial Intelligence and Logic Programming*, volume III, pages 353–395. Oxford University Press, 1994.

[19] H. Prakken. A Study of Accrual of Arguments, with Applications to Evidential Reasoning. In *Proc ICAIL 2005*, pages 85–94, New York, NY, USA, 2005. ACM.

[20] H. Prakken. An abstract framework for argumentation with structured arguments. *Argument & Computation*, 1(2):93–124, 2010.

[21] H. Prakken and S. Modgil. Clarifying some misconceptions on the ASPIC$^+$ framework. In B. Verheij, S. Szeider, and S. Woltran, editors, *Proc COMMA 2012*, pages 442–453. Vienna, Austria, Sept. 2012.